

Super-resolution wavefront reconstruction



Subtitle: break free from aligned grids

Sylvain Oberti

*On behalf of Carlos Correia, Pierre Guiraud,
Guido Agapito, Thierry Fusco, Johann Kolb, Benoit Neichel, Jesse Cranney, Marcos Van Dam*

■ Presentation of recent A&A paper

- Definition and history
- Mathematical formalism in 1D
- Application to Shack-Hartmann
- Sensitivity analysis
- Numerical application

Astronomy & Astrophysics manuscript no. aa43954-22
September 9, 2022

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Received 4 May 2022 / Accepted 2 August 2022

■ On sky verification with AOF

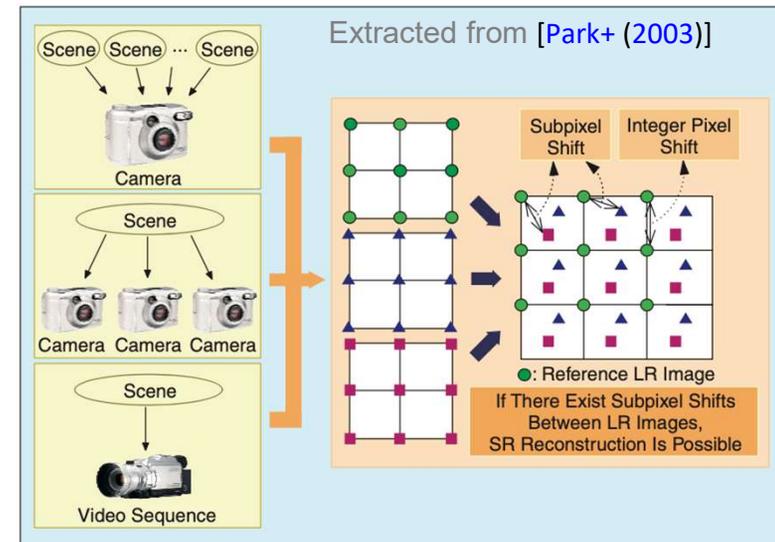
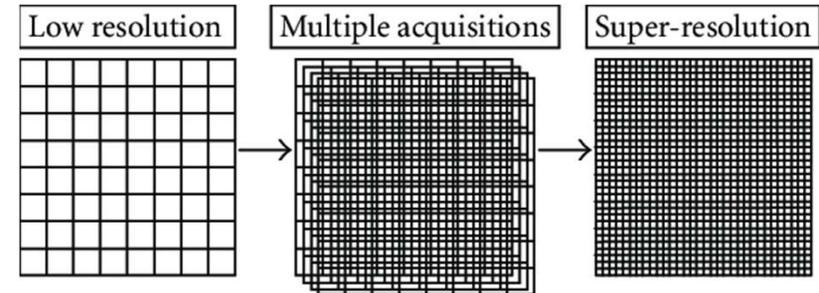
■ Application to future AO designs: MORFEO, MAVIS, HARMONI LTAO

■ Super-resolution applied to the Pyramid WFS

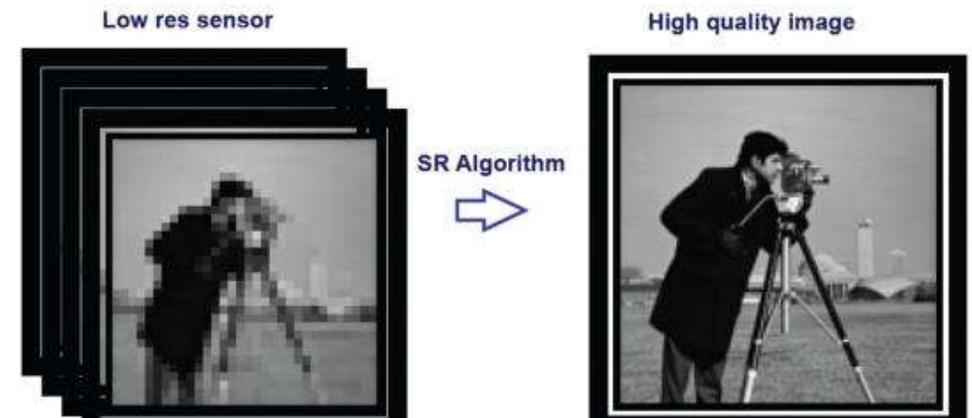
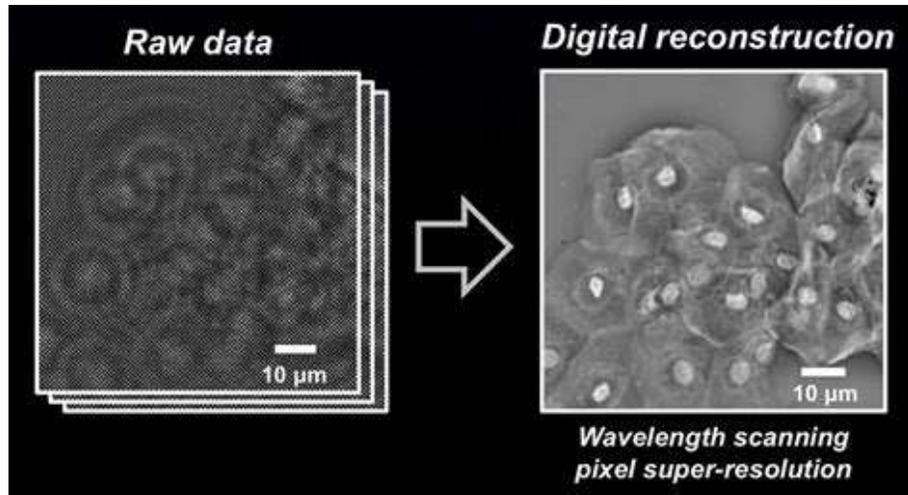
■ Future steps

Definition and history of SR

- The term *Super-Resolution* (SR) as is used here refers to the application of signal processing techniques to obtain higher-resolution (HR) data products from multiple low-resolution (LR) samples
- First work on “image upscaling” [Tsai+ (1984)]
- The term “Super-resolution” itself appeared in [Irani+ (1990)]
- SR techniques are now relatively standard in vision science whenever imagery data is available with sub-pixel shifts between LR samples [Park (2003)]
- Reconstruction a posteriori of a temporal sequence of LR measurements from a single WFS, with fractional subaperture shifts applied to each temporal sample [Bara+ (2013)]
- Application to wavefront reconstruction for tomographic AO suggested by [Oberti, AO4ELT5 (2017)]



Super-resolution imaging: classical method

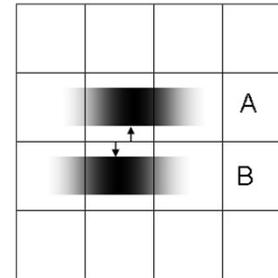


Ultimate limit: diffraction

Super-resolution in astronomy

- “Drizzle” method to enhance the resolution of under-sampled HST images [[Fruchter & Hook \(2002\)](#)] where the high spatial frequency information of the HST images are recovered by combining sub-pixel dithered images.

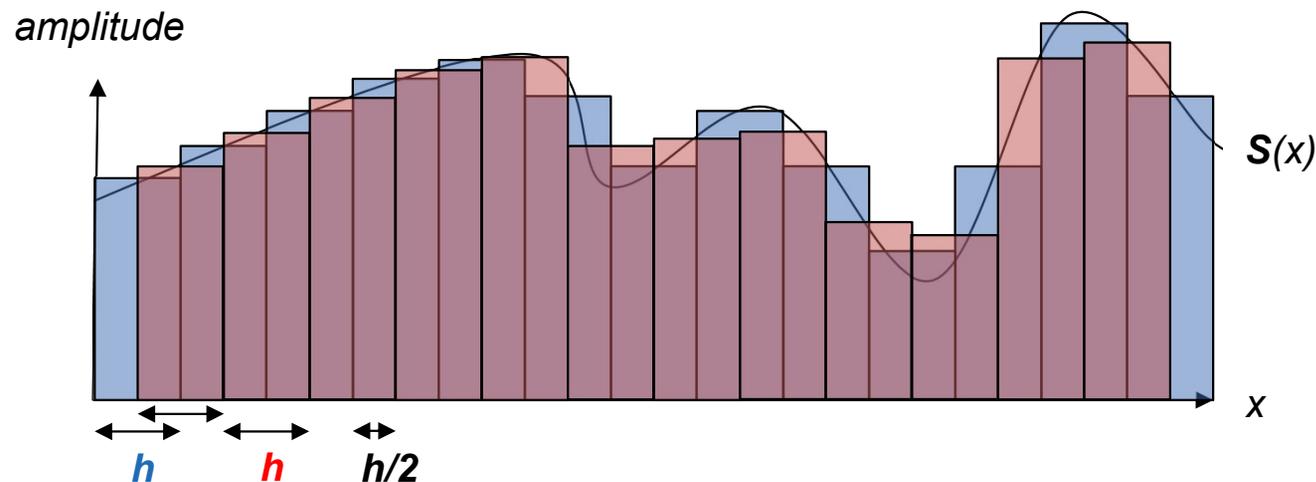
- Spot size determination by scanning under-sampled images



- GALACSI NFM: LTAO reconstruction with misregistrations in model [[Oberti+ \(2016 & 2018\)](#)]
- NAOMI DM flattening from shifted low order WFS data sequence [[Woillez+ \(2019\)](#)]
- **“Statistical” super-resolution**: minimum residual variance reconstruction. Using some sort of regularization, the reconstructed wavefront can be extrapolated to larger spatial frequencies
 - Minimum Mean Square Error reconstructor: [[Fusco+ \(2001\)](#)]; [Conan \(2014\)](#); [Correia+ \(2014\)](#)]
 - Tomography oversampling with a factor of 2: [[Ellerbroek & Rigaut \(2001\)](#)]; [Wang+ \(2012\)](#)]
- Let us now focus on the **“geometric” super-resolution**

Basic mono-dimensional description

Let S be a function of the wavefront along one axis (a line across the pupil). S can be the phase, its first or second derivative or an intensity pattern at the focal plane etc



Sampling with a grid of sampling period h and sampling element of size h

Sampling with same resolution but grid shifted by half a sampling element

Combining the 2 sampling patterns (blue and red), we effectively get a twice smaller sampling period $h/2$ but keeping a “subaperture” size of h

Periodic Nonuniform Sampling (PNS) in 1D

- Shifting N sampling grids by a fraction of h leads to a meta sampling grid:

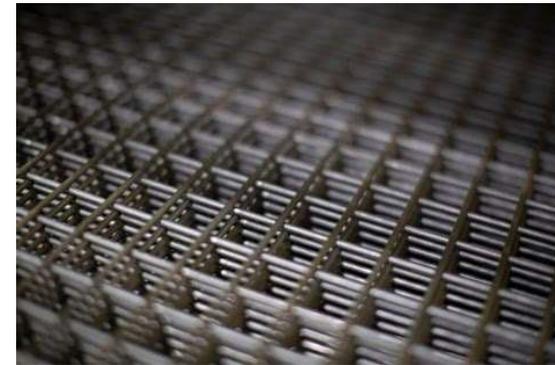
$$\mathbf{s}_{N,h}(x) = (\bar{\mathbf{s}}_h \cdot \text{III}_{\frac{h}{N}})(x) = (\mathbf{s} * \chi_h) \cdot \text{III}_{\frac{h}{N}}(x)$$

- In Fourier, we get:

$$\hat{\mathbf{s}}_{N,h}(k) = h\hat{\mathbf{s}}(k) \text{sinc}(\pi hk) * \text{III}_{\frac{N}{h}}(k)$$

Sensitivity function remains the same
Only related to subaperture size h

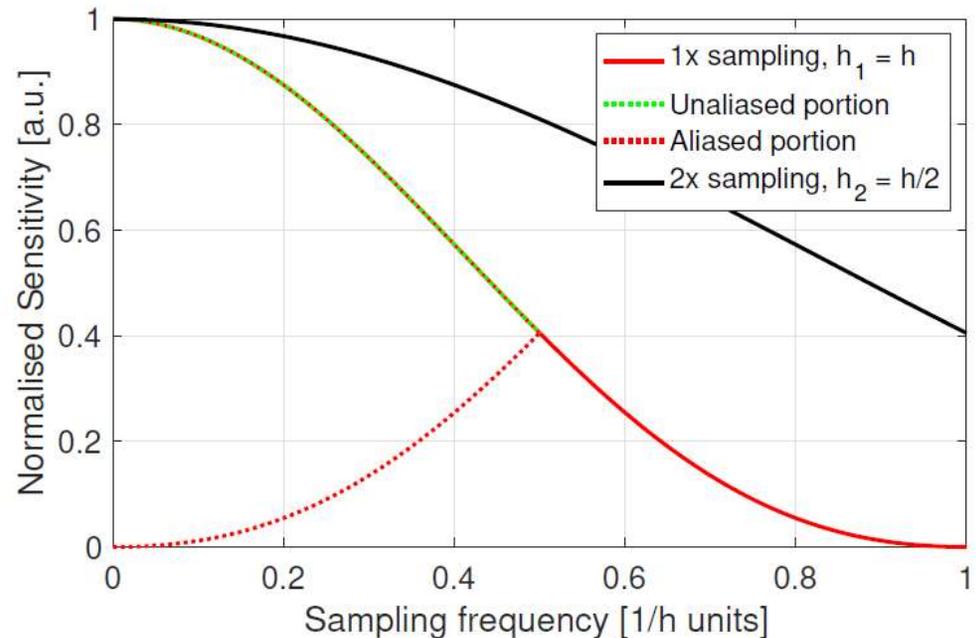
Higher resolution sampling and aliasing reduction with Dirac comb of period N/h



When designing multi WFS AO systems, we can **decouple** the **size of the measurement cell h** from the **sampling period**. This is a game changer for multi WFS AO systems !

Dealiasing: sensitivity function

- **Dotted red curve:** sampling grid of step h
- **Solid red curve:** super-resolved combination of sampling grids of step h with a SR factor of 2
- **Black curve:** sampling grid of step $h/2$



- SR factor of 2 allows dealiasing up to spatial frequency $1/h$, the first null of the *sinc* function
- Noise propagation under SR is to be closely looked at

Reconstruction and non-uniform sampling

- PNS particular case: **meta-uniform sampling**

- $\hat{S}_{N,h}(k) = h\hat{S}(k)\text{sinc}(\pi hk) * \text{III}_{\frac{N}{h}}(k)$
- Nyquist-Shannon theorem: perfect reconstruction up to the frequency $k_{max} = N/2h$ without aliasing.
- **Super-resolution factor** $F = k_{max}/2h = N$, i.e. the number of sampling grids
- Meta-uniform sampling → optimal sampling scheme to minimize number of grids. Ex: 4 grids to reach $F=2$

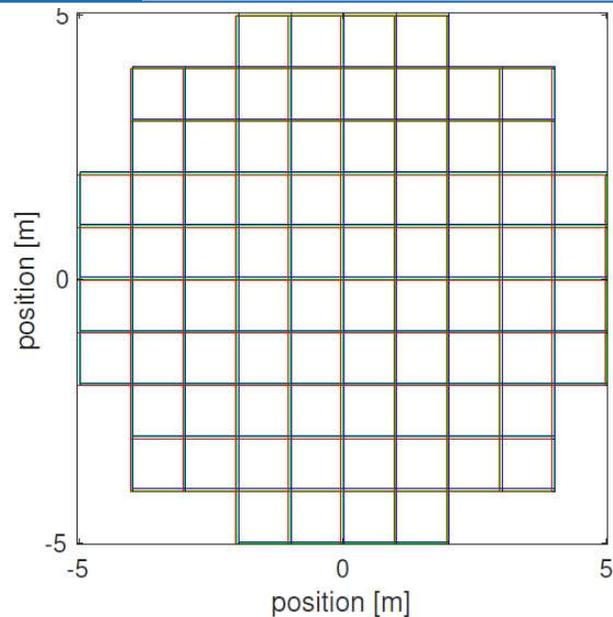
- **Non-uniform sampling.** The Paley–Wiener–Levinson theorem shows that a band limited signal can be reconstructed if the 2 following conditions are satisfied:

- The **average sampling rate** is larger than the Nyquist frequency of interest [Beutler (1966)] $k_e = 1/h_e \geq 2k_{max}$
- The **Kadec condition** [Kadec (1964)] states that the nonuniform sampling shall be constructed by relocating the points of a uniform sampling of step $1/2k_{max}$ no further than $1/8k_{max}$ from their original location.

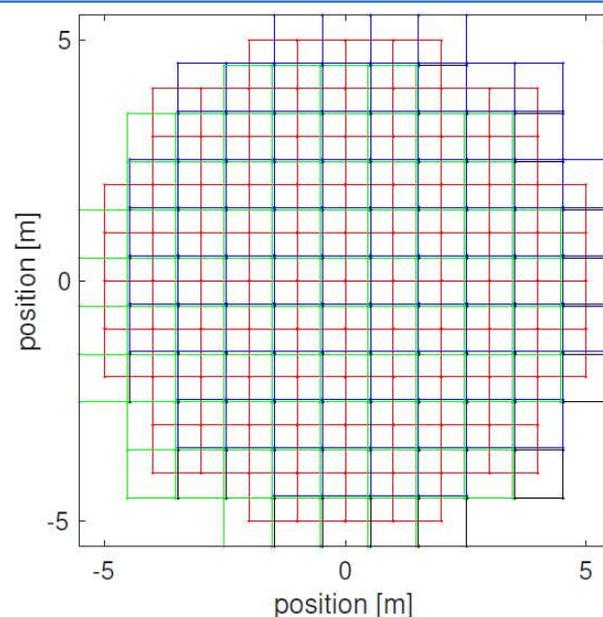


■ **Take home message:** In the PNS framework, the N sampling grids' offsets shall be close to but not necessarily equal to a multiple of h/N , in order to achieve optimal SR ($F = N$)

Bi-dimensional model for a Shack-Hartmann WFS



4 co-aligned sampling grids



Meta-uniform sampling with 4 grids shifted by $h/2$

With

- \mathcal{G} the phase-to-slopes linear operator
- d the subaperture width
- h the sampling step

Individual SH WFS

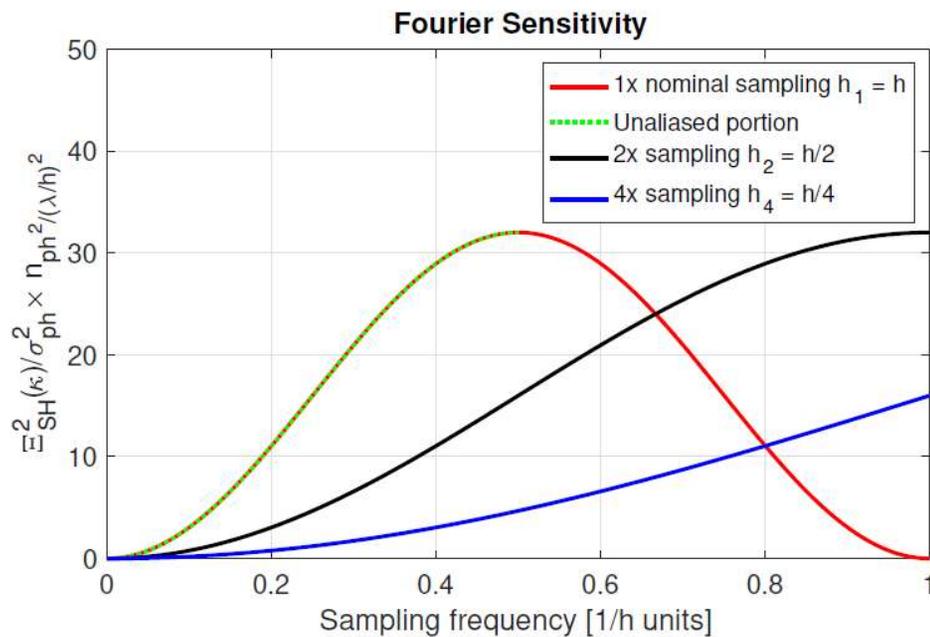
$$\mathcal{G} = \text{III}\left(\frac{\mathbf{x}}{h}\right) \times \left[\Pi\left(\frac{\mathbf{x} - 1/2}{d}\right) \otimes \nabla \right]$$

Combination of 4 SH WFS

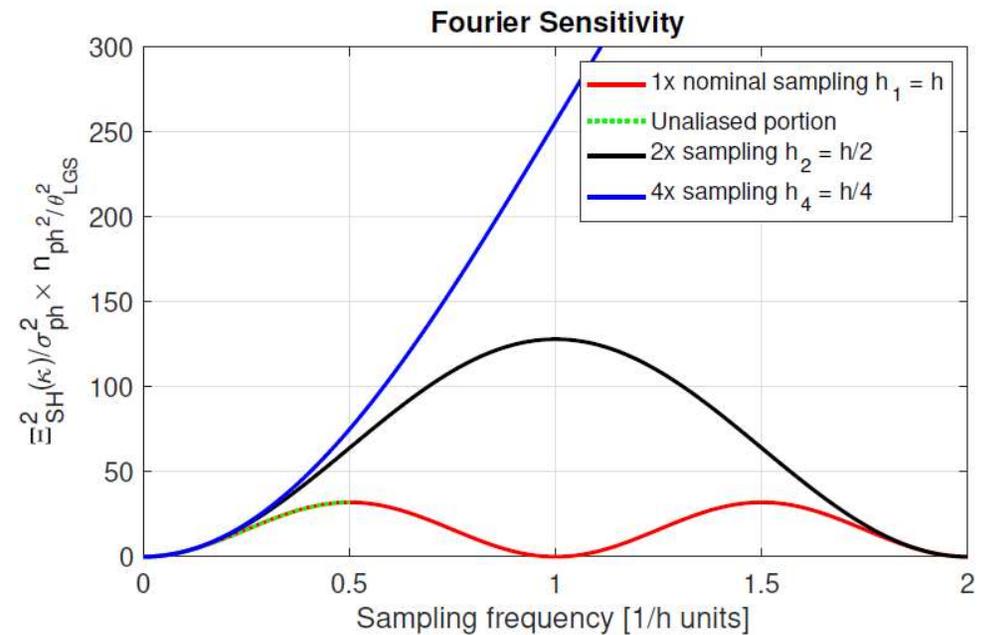
$$\tilde{\mathcal{G}} = \text{III}\left(\frac{\mathbf{x}}{h/2}\right) \times \left[\Pi\left(\frac{\mathbf{x} - 1/2}{h}\right) \otimes \nabla \right]$$

■ This 2D formalism describes the meta-uniform sampling obtained by combining 4 SH WFS

Fourier sensitivity for SH WFS



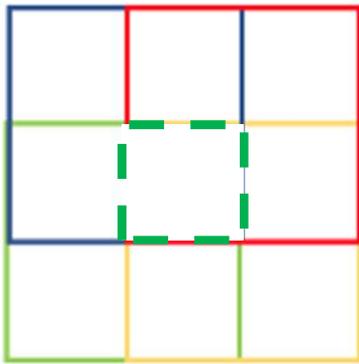
Diffraction limited case



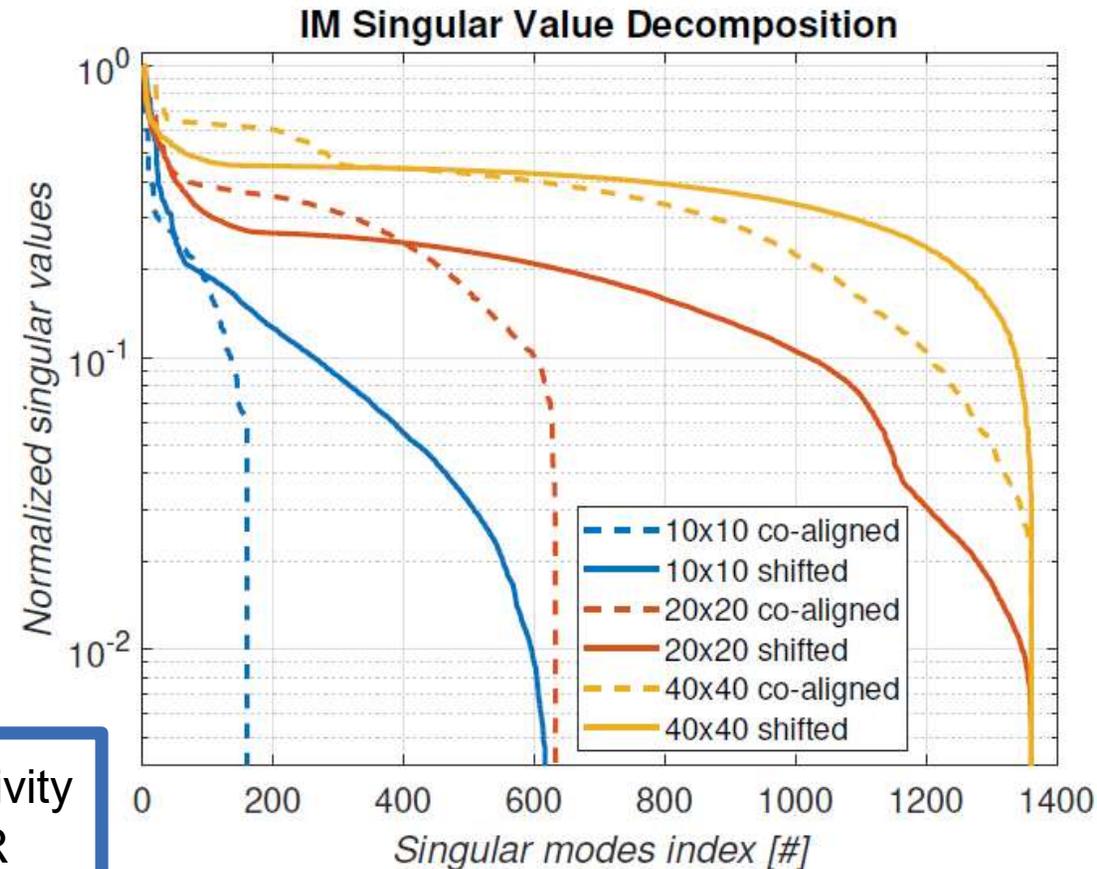
Seeing limited or LGS spot sensing case

SR enhances the sensitivity to lower spatial frequencies in the diffraction limited case and reduces it with extended sources

Numerical application to multi WFS AO

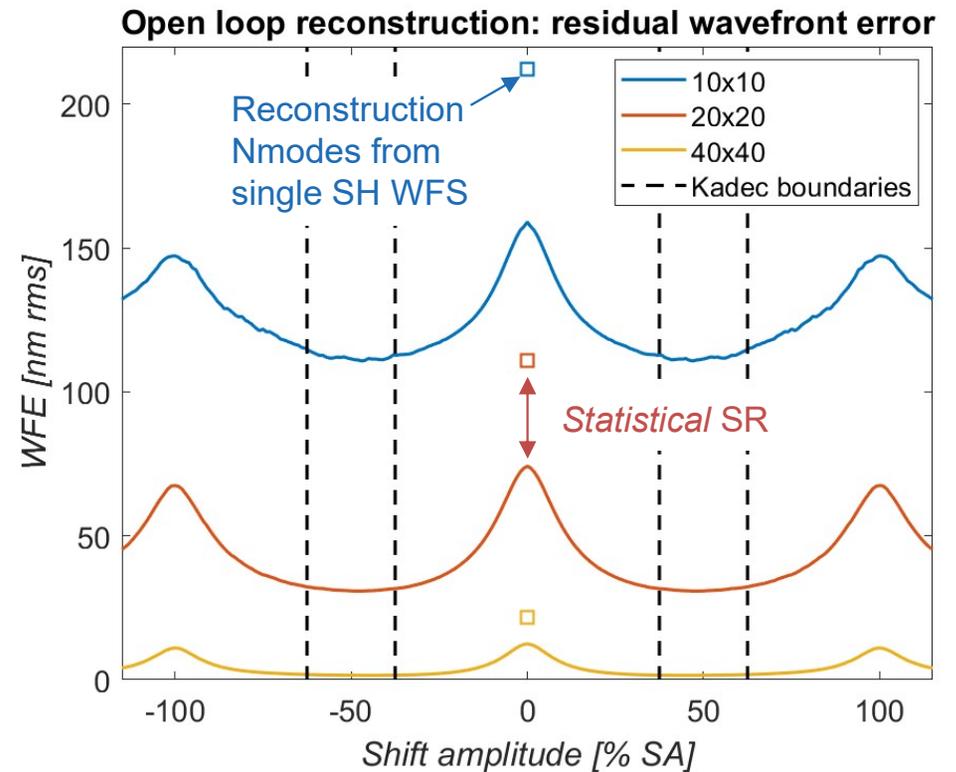
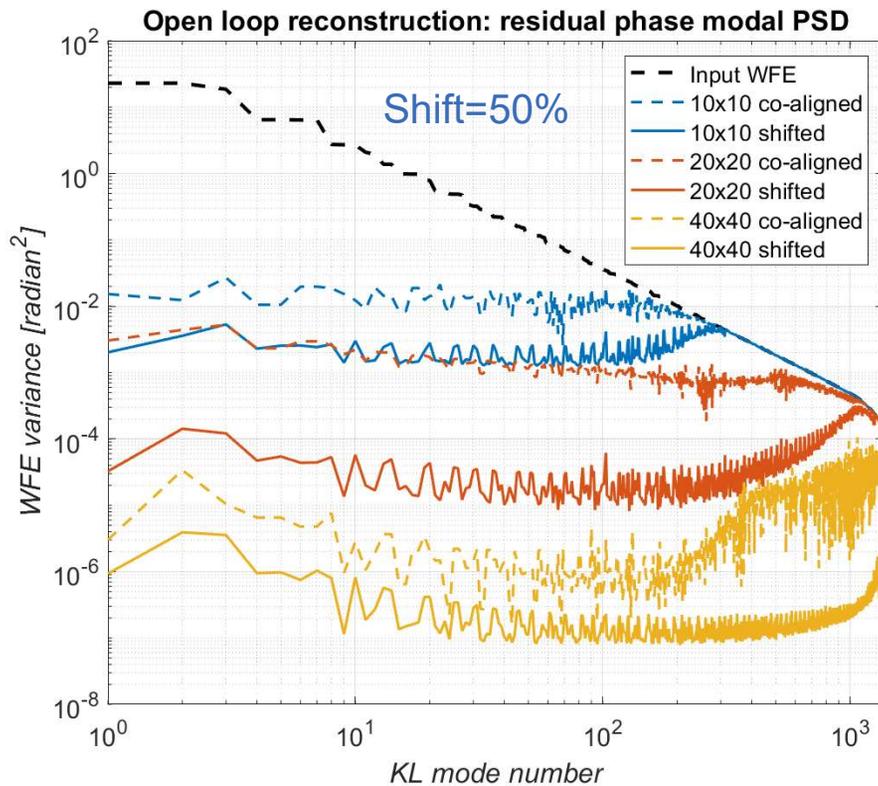


Telescope Diameter	8 m
Number of WFSs	4 SH-WFS
Native resolution case	$n_{SA} = 40$ subapertures
Lower resolution cases	$n_{SA} = 20$ or 10 subapertures
Turbulent wavefront	1 layer in the pupil plane: KL modes



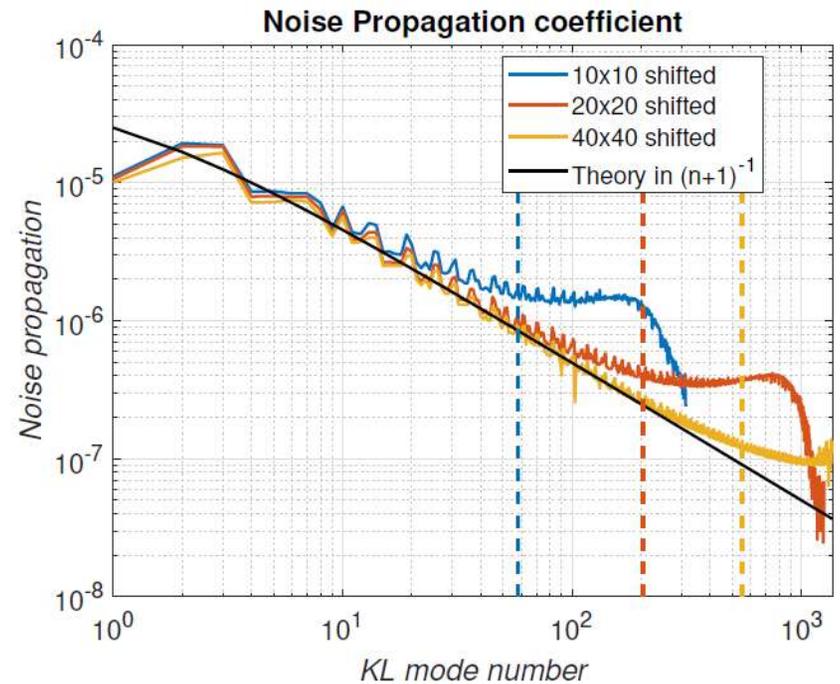
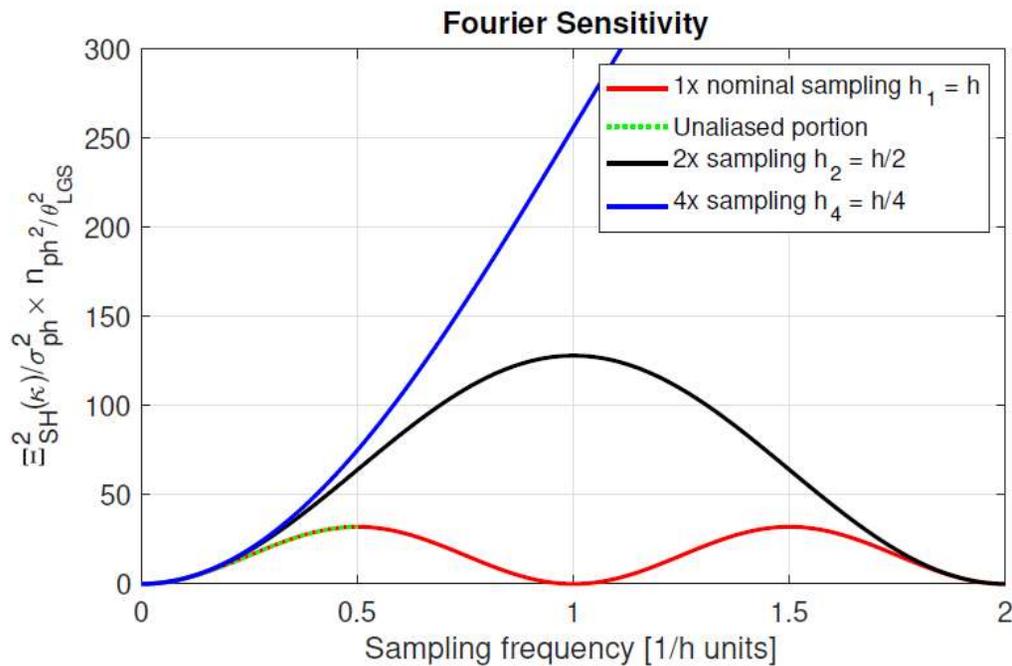
■ The IM SVD highlights the improved sensitivity to higher spatial frequencies provided by SR

Open loop reconstruction



Geometrical SR kicks in already with tiny shifts and exhibits a robust optimum

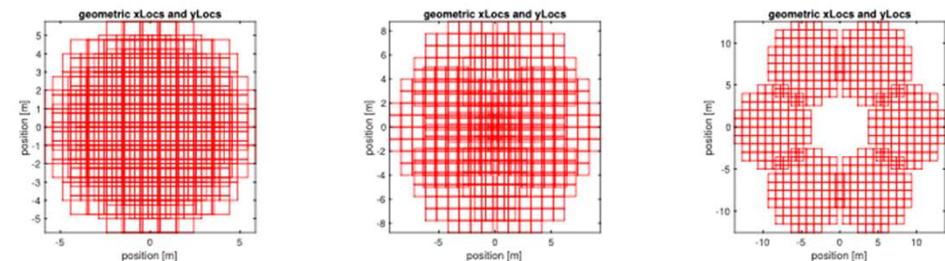
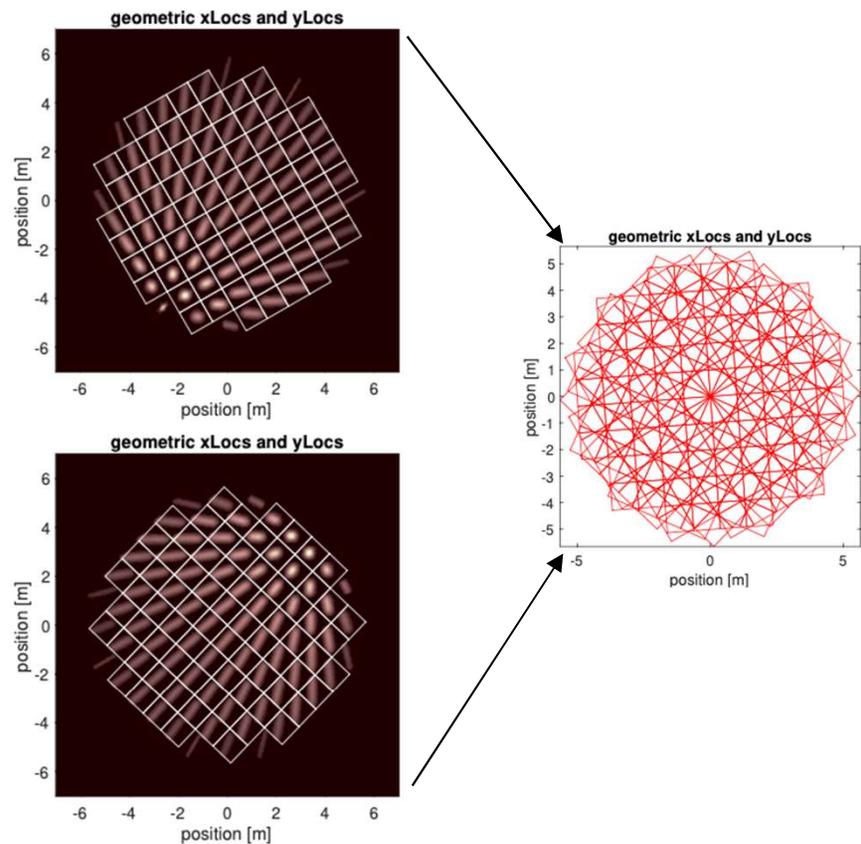
Noise propagation



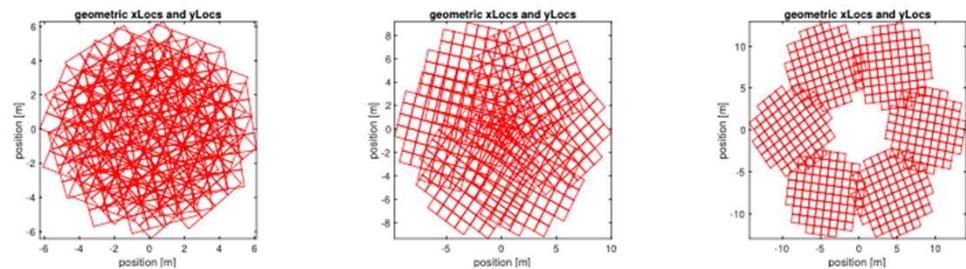
■ Noise propagation is not affected by SR across low spatial frequencies until it reaches a plateau towards the first null of the sinc function



Break free from regular, symmetric, homogeneous sampling



Regular 6 LGS case at three different altitudes



Differentially rotated 6 LGS case at three different altitudes

Generate diversity of information and use it





Super-resolution wavefront reconstruction

■ Presentation of recent A&A paper

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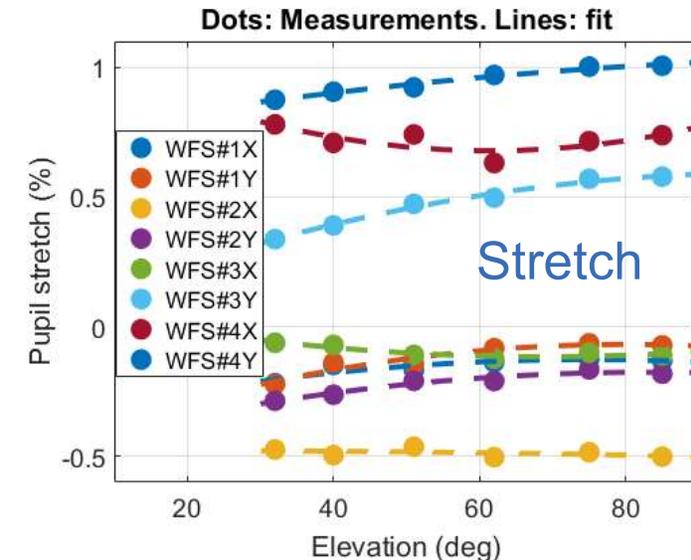
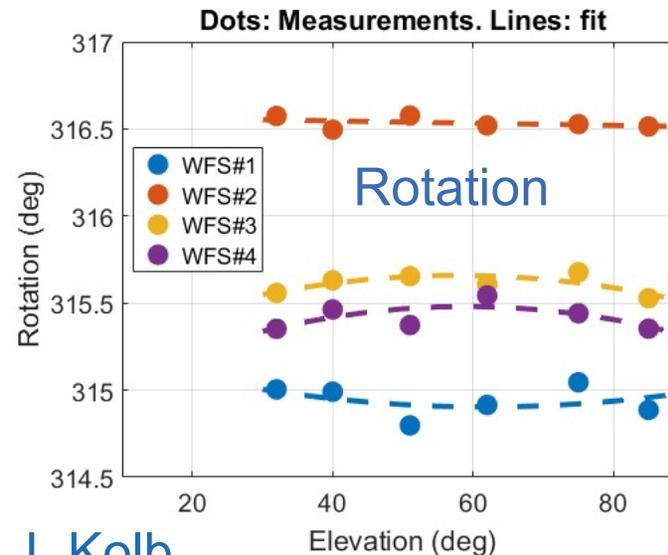
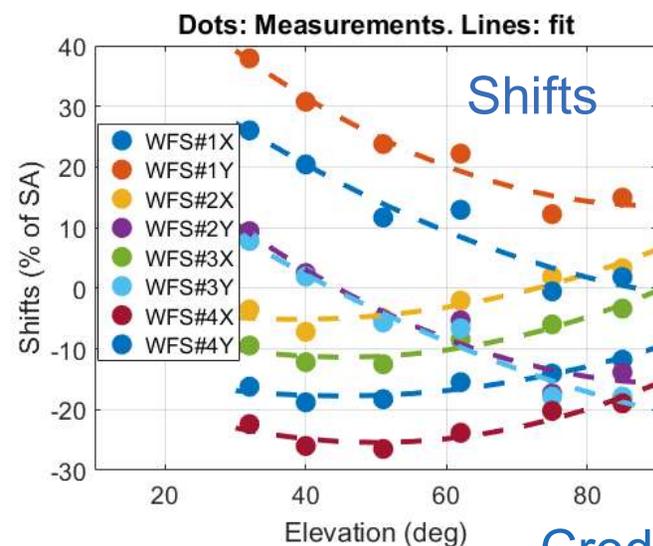
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■ Next steps



GALACSI NFM: differential registrations



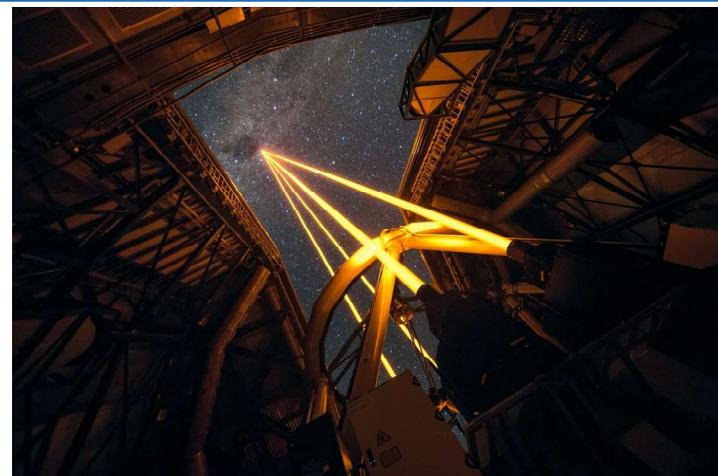
Credit: J. Kolb

- GALACSI NFM takes misregistrations into account in the PSIM model
- Injected knowledge into model enables SR through MMSE reconstruction
- 1300 modes are reconstructed in each layer. Then, projected on the 1156 DoF of the DM, filtered to 850 KL modes

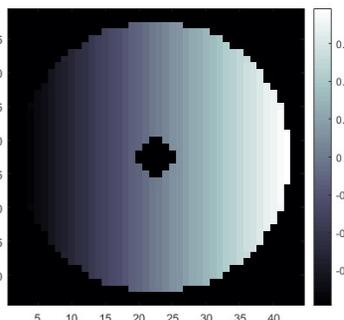


On sky experiment: Binning LGS WFS SH slopes

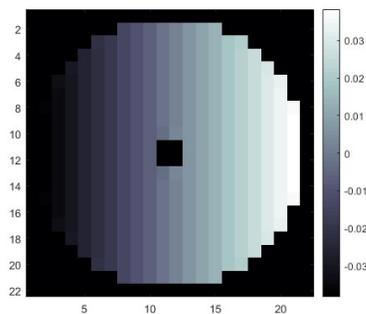
GALACSI NFM



40x40

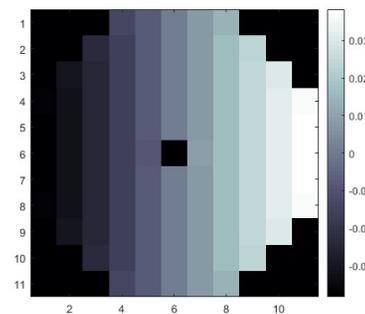


20x20
binning

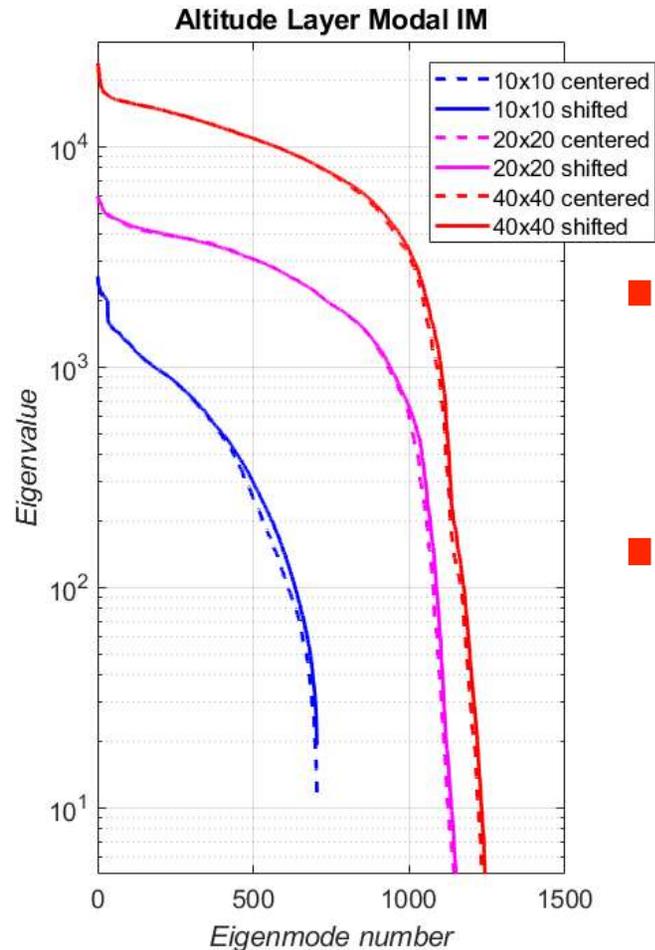
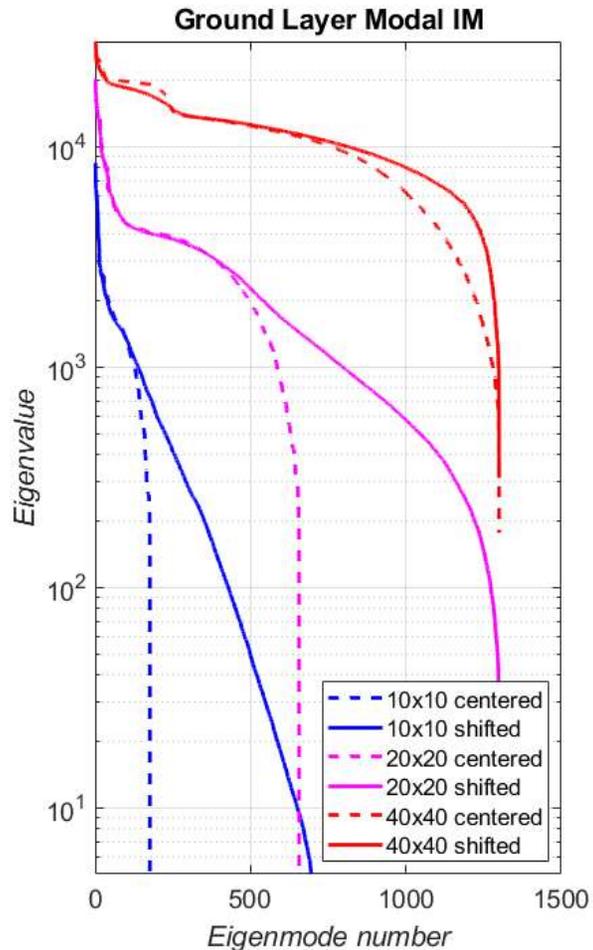


Projection matrix plugged into CM

10x10
binning



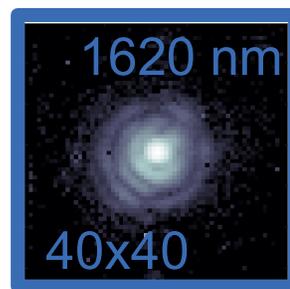
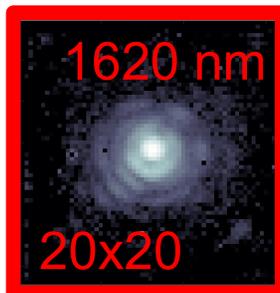
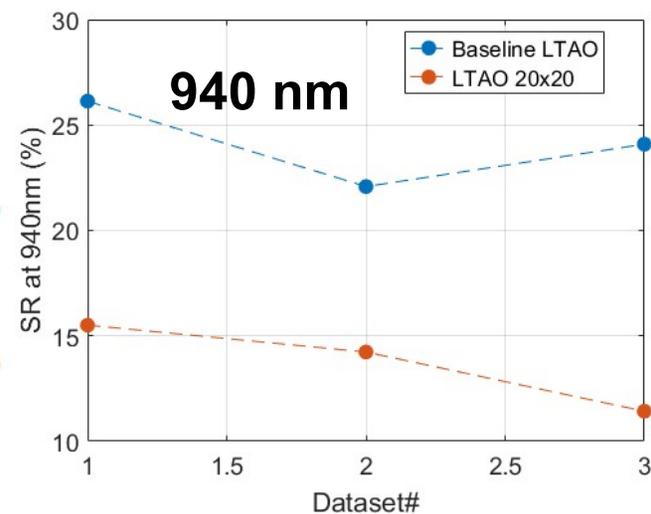
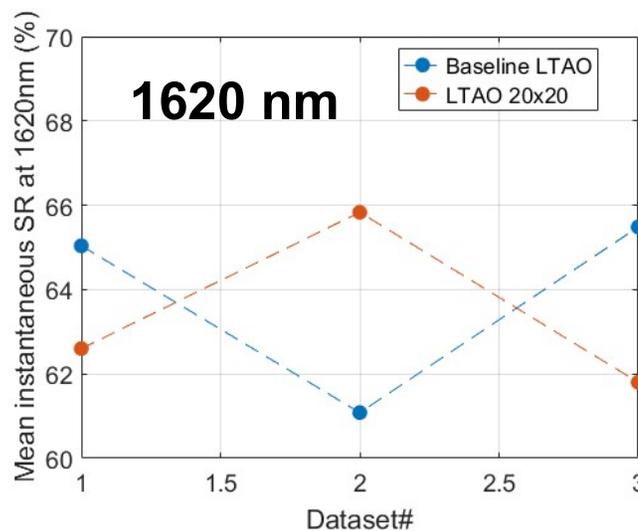
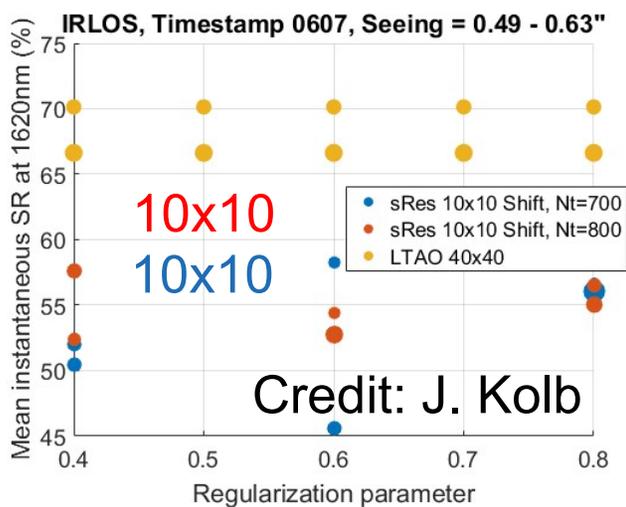
System sensitivity to additional DoF



- In the pupil plane
 - Clear advantage to apply super-resolution to 20x20 or 10x10 SA
 - For native HO resolution (40x40), aliasing reduction is visible on the least seen modes
- In altitude conjugated planes
 - As expected, the super-resolution is already built in



On sky demonstration with GALACSI NFM



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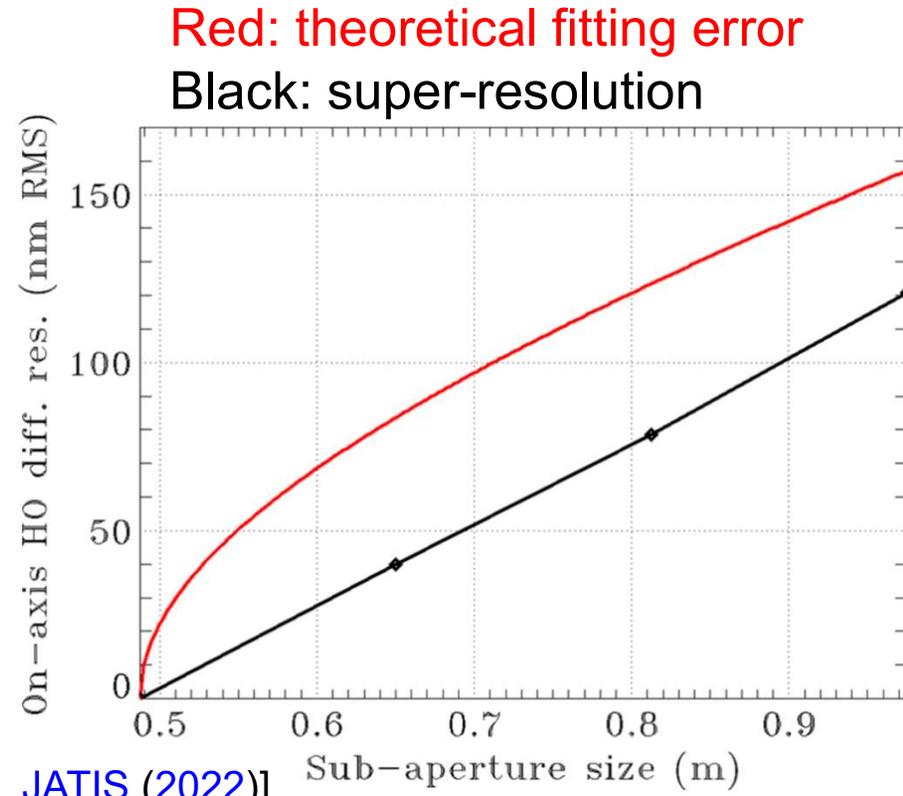
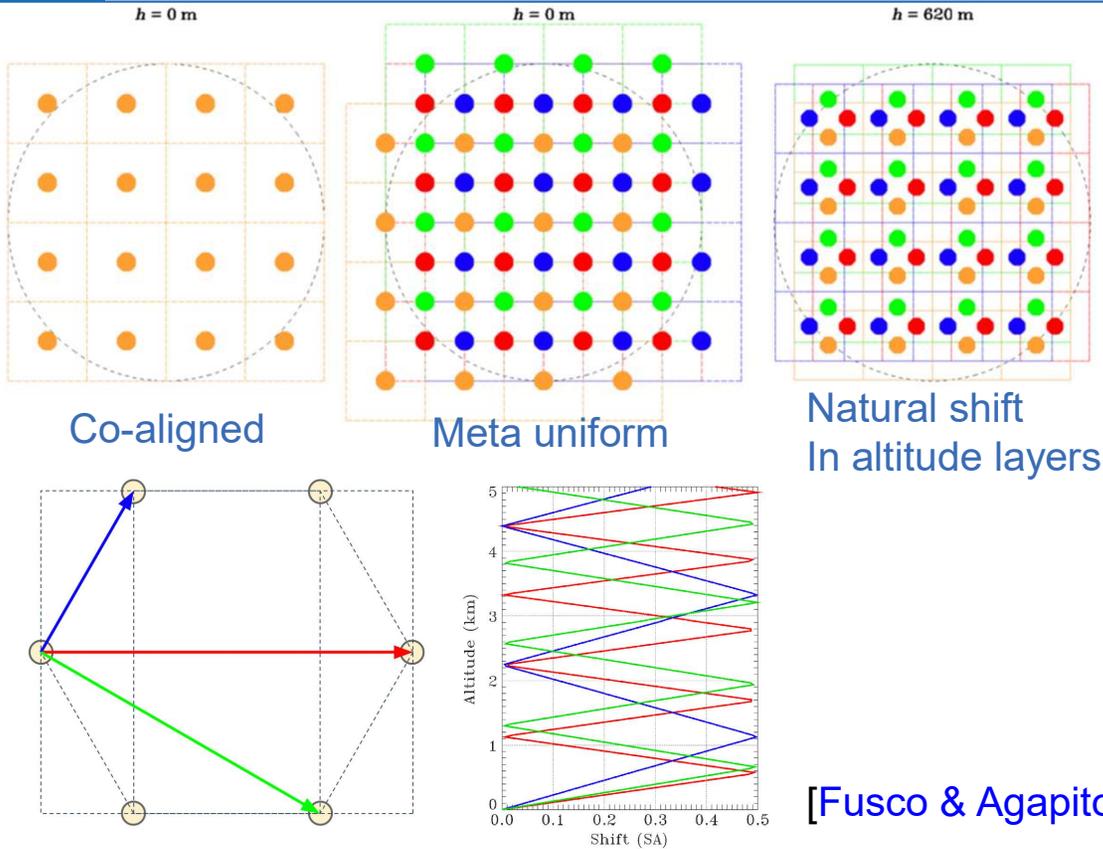
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MORFEO: naturally induced SR



[Fusco & Agapito+, JATIS (2022)]

1st rule of SR: allow to reconstruct larger spatial frequencies than 1 WFS can sample

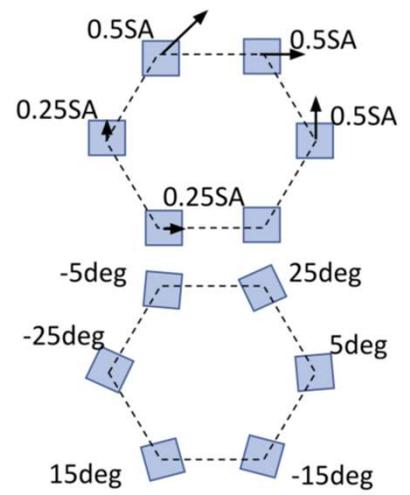
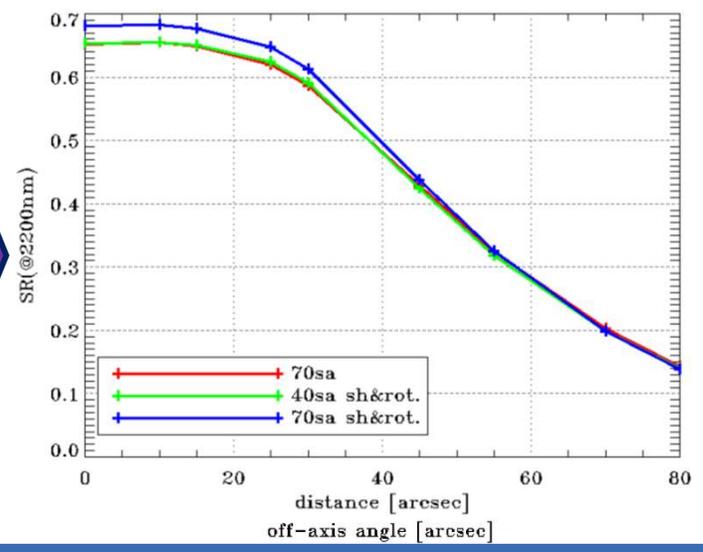
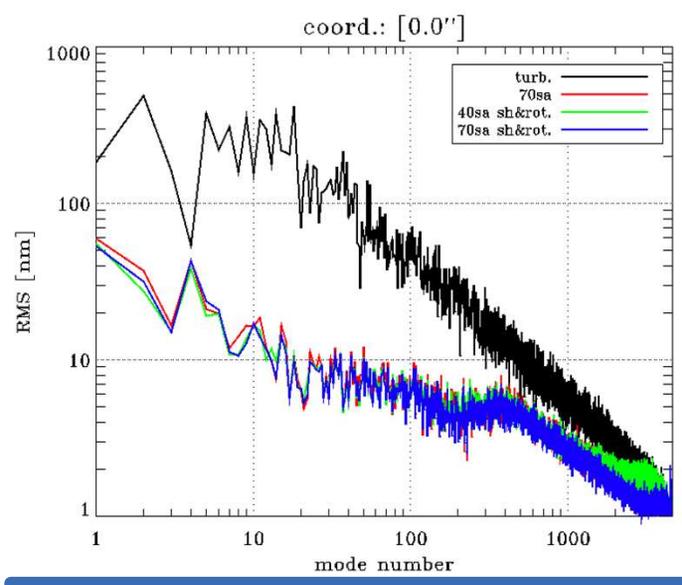


MORFEO: deliberately introduced SR



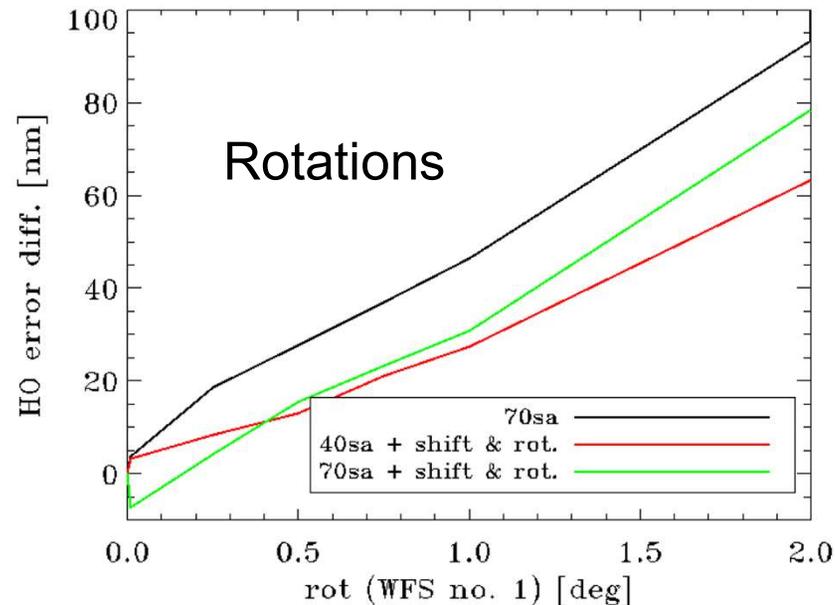
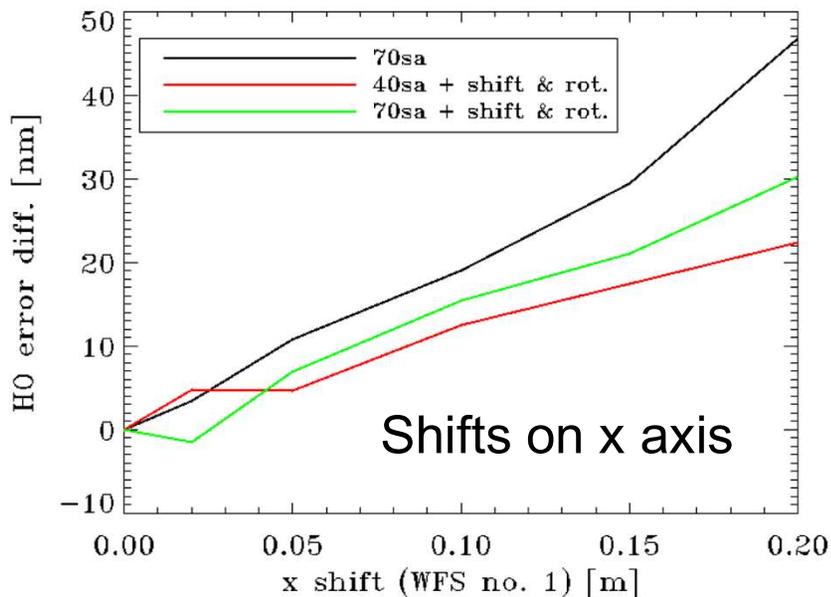
- 70x70 SA → 193nm (mode 2-4500) on axis
- 40x40 SA, Rotation and Shift → 193nm (mode 2-4500) on axis
- 70x70 SA, Rotation and Shift → 179nm (mode 2-4500) on axis

[Agapito+, SPIE (2020)]



- Flavor 1: Reduce number of SA while preserving a similar level of performance
- Flavor 2: Reduce aliasing at iso number of SA





Unstable loops:

- 70x70SA for values $> 0.2\text{m}$ (36% of a SA) [Agapito+, SPIE (2020)]
- 70x70SA with shifts and rotations for values $> 0.4\text{m}$ (72% of a SA)
- 40x40SA with shifts and rotations is stable for all tested values ($\leq 0.60\text{m}$, $\leq 62\%$ of a SA)

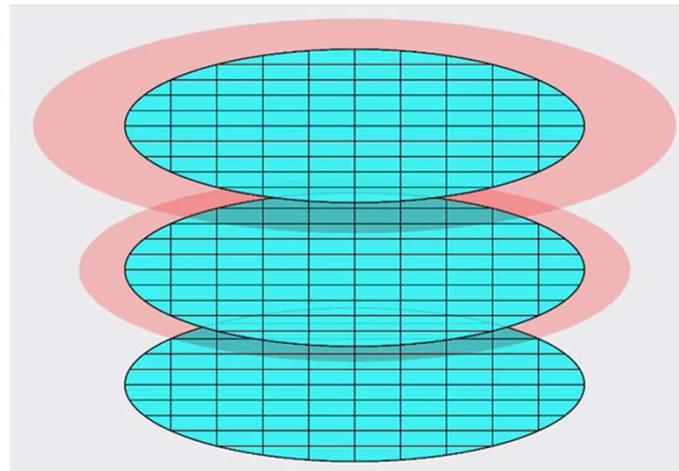
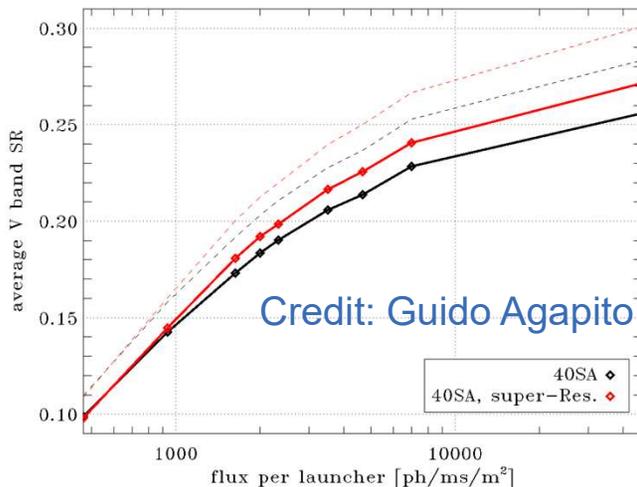
- SR more robust to model errors than the classical AO case
- With SR, it is also more robust to have less SA



SR : the baseline for MAVIS



- An improvement of 25nm rms is expected from SR with 40x40 SA
- MAVIS adopts super-resolution as baseline:
 - Several SR geometries investigated
 - Cost function to evaluate them
- Number of SA chosen considering super-resolution, LGS elongation orientation and other practical aspects [see Talk from Cedric Plantet / Jesse Cranney](#)



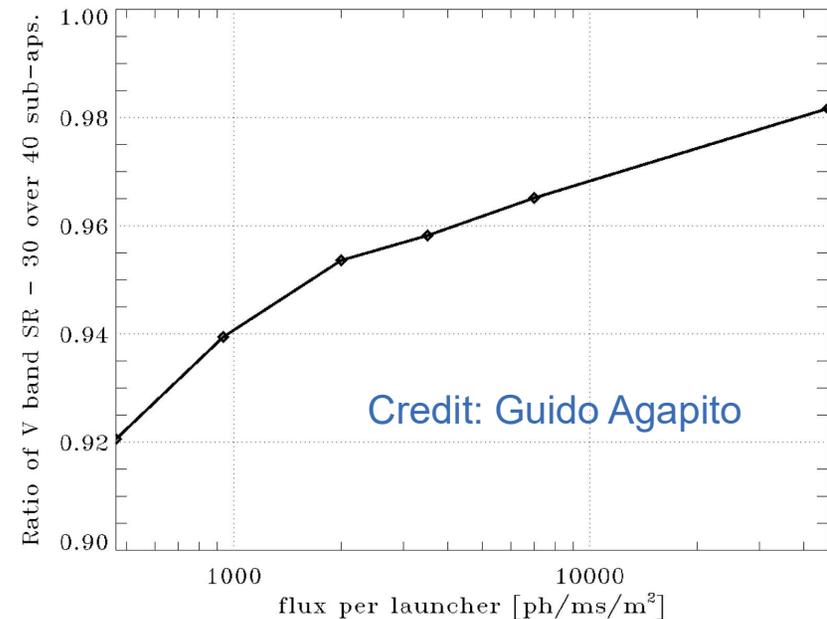
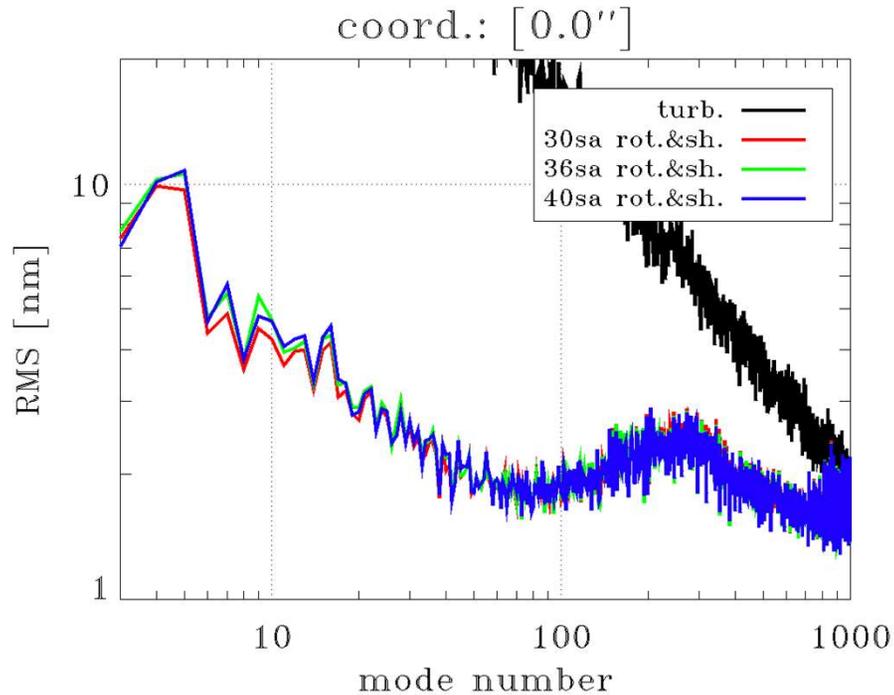
[Cranney+ (2021)]

TABLE I
OPTIMISATION AND END-TO-END SIMULATION RESULTS FOR EMPIRICAL (*) AND OPTIMISED SUPER-RESOLUTION GEOMETRIES.

Super-res Geometry Used	Cost Evaluation	Wave-front Error
(*) No Super-resolution	0.6214	74.23 nm
(*) $\theta_j = 11.25^\circ \times j$	0.5164	65.96 nm
Random Initial Value	0.5948	69.03 nm
Optimised (all params free)	0.4896	66.80 nm
Optimised (only θ_i 's free)	0.5164	65.61 nm



Number of subapertures



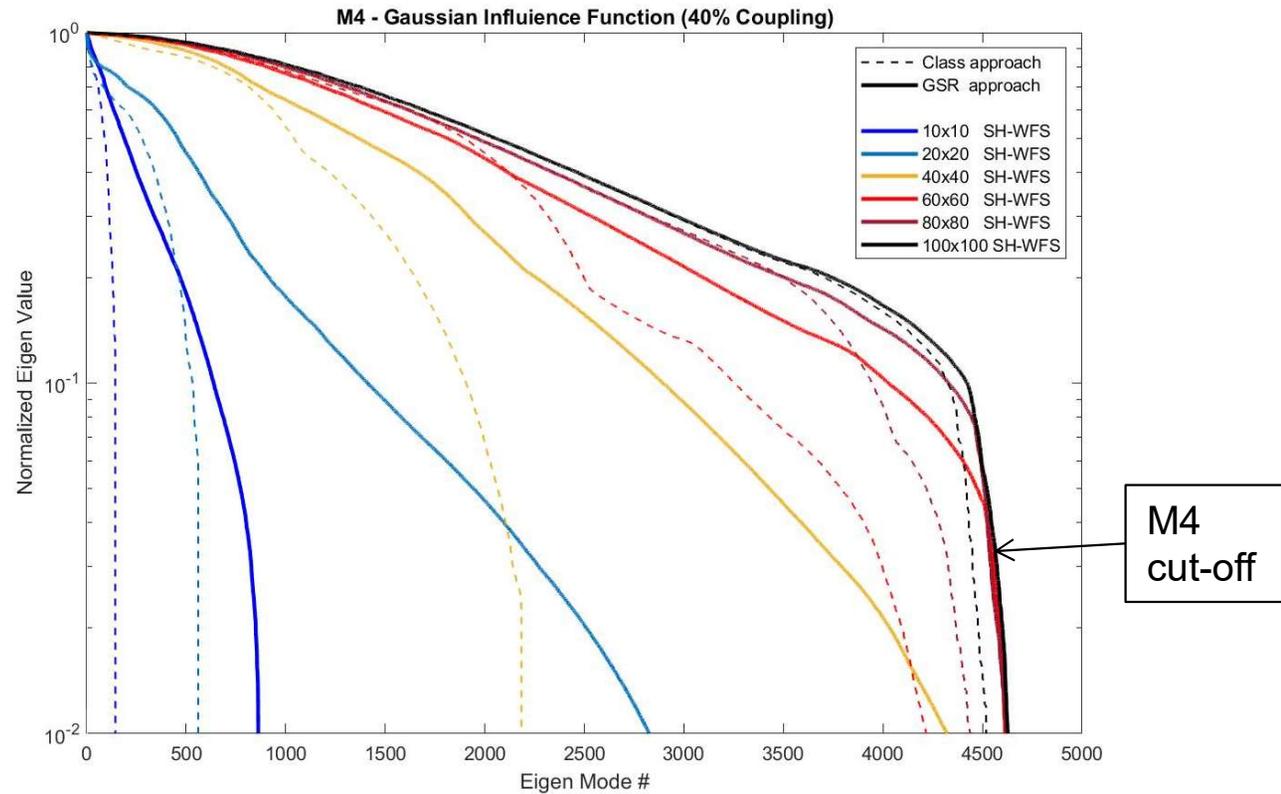
- 30×30 case has an advantage on low order modes because it has a large FoV (8×8 pixel/SA)
- In median flux 40×40 case has an advantage of 10nm rms

- 40×40 case advantage is larger at low flux → lower noise propagation
- Because of 8x8 pixels per SA in 30x30, with additional read-out noise

■ Low dependency between performance and number of SA

SR for HARMONI LTAO: IM SVD

Credit: Thierry Fusco



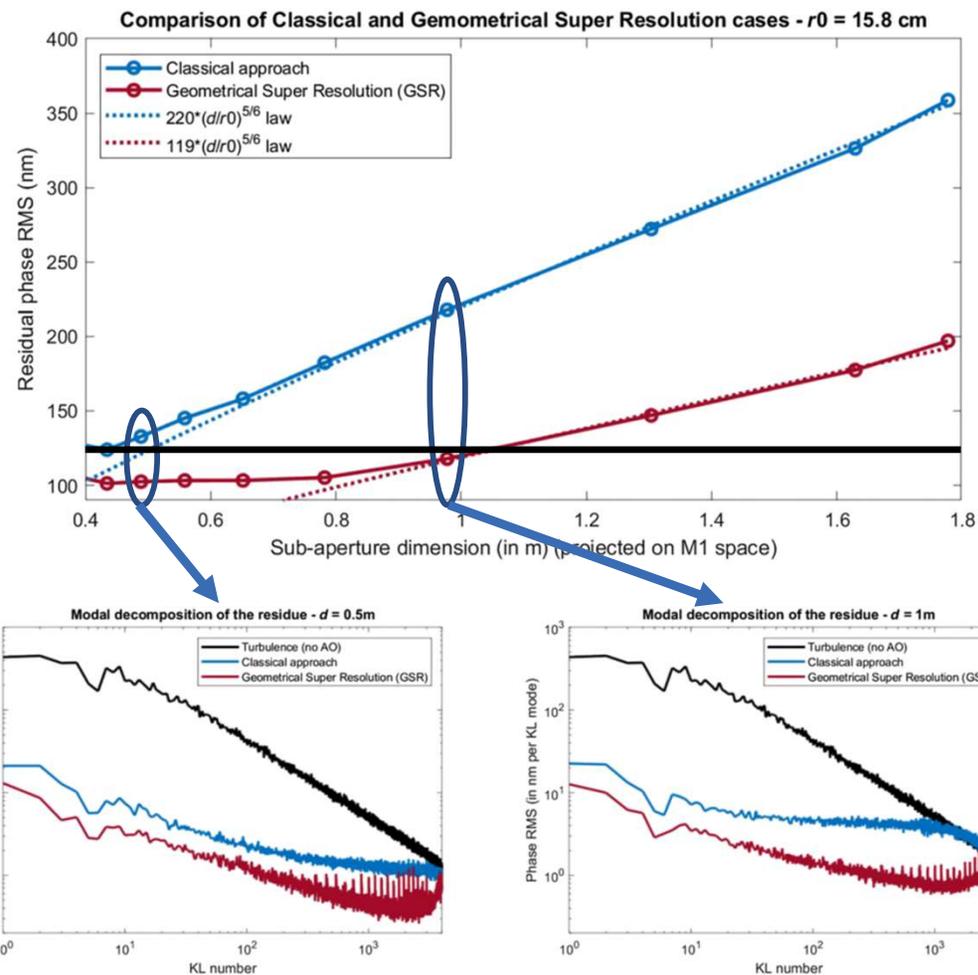
- 60x60 SA allow reaching the M4 cut-off frequency
- With 40x40 SA, 4000 modes are visible to the system

SR for HARMONI LTAO: WFE

[Fusco & Agapito+, JATIS (2022)]

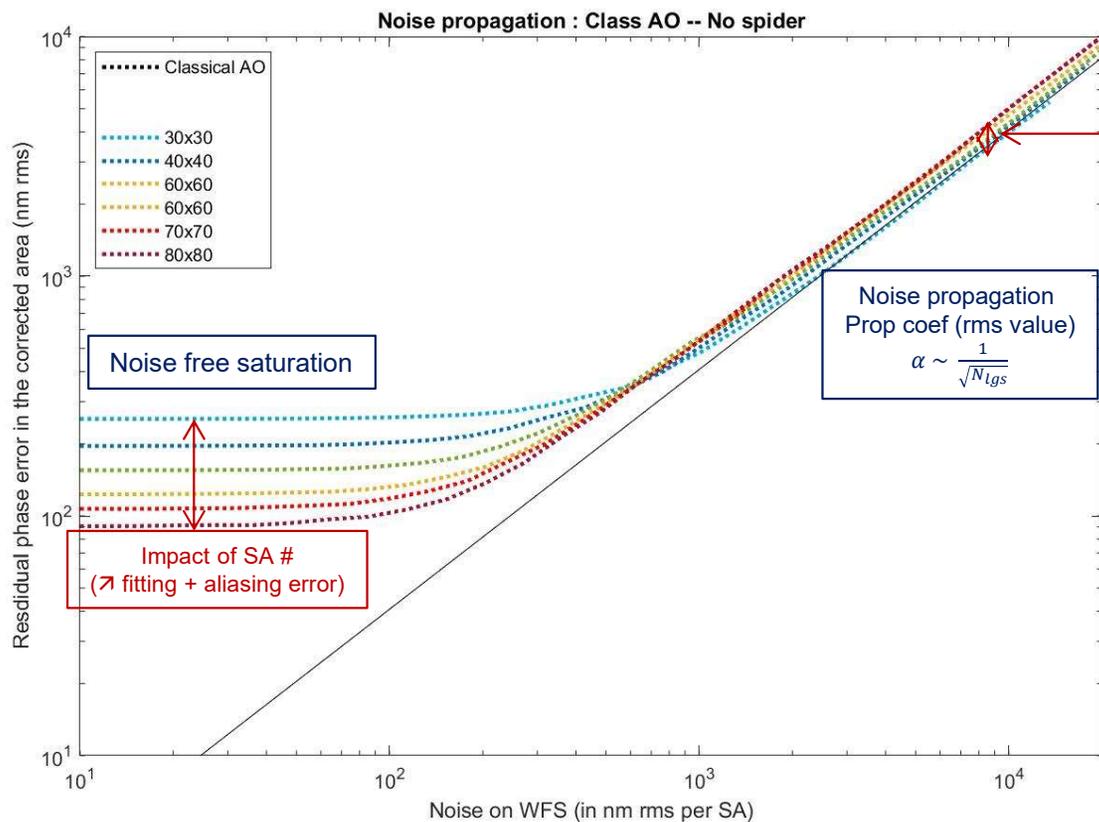
- 1 single turb layer
- 100 independent phase screens in the pupil ($h=0$)
- $r_0@0.5\mu\text{m} = 15.7 \text{ cm}$
- Seeing = $0.66''$
- $L_0 = 25 \text{ m}$

- Residual phase follows a $(d/r_0)^{5/6}$ law for classical AO system
- With SR also, but the WFE reaches a plateau for SA sizes $< 1\text{m}$
- The minimal error is reached with a SA of 0.4 m with the classical approach. The same performance can be achieved with SA width $> 1\text{m}$ with SR (black line)
- The larger the SA the larger the SR benefit



H-LTAO: Noise propagation for class AO

M4 – IF Gauss
WFS Geo / 6 LGS
1 layer in the pupil

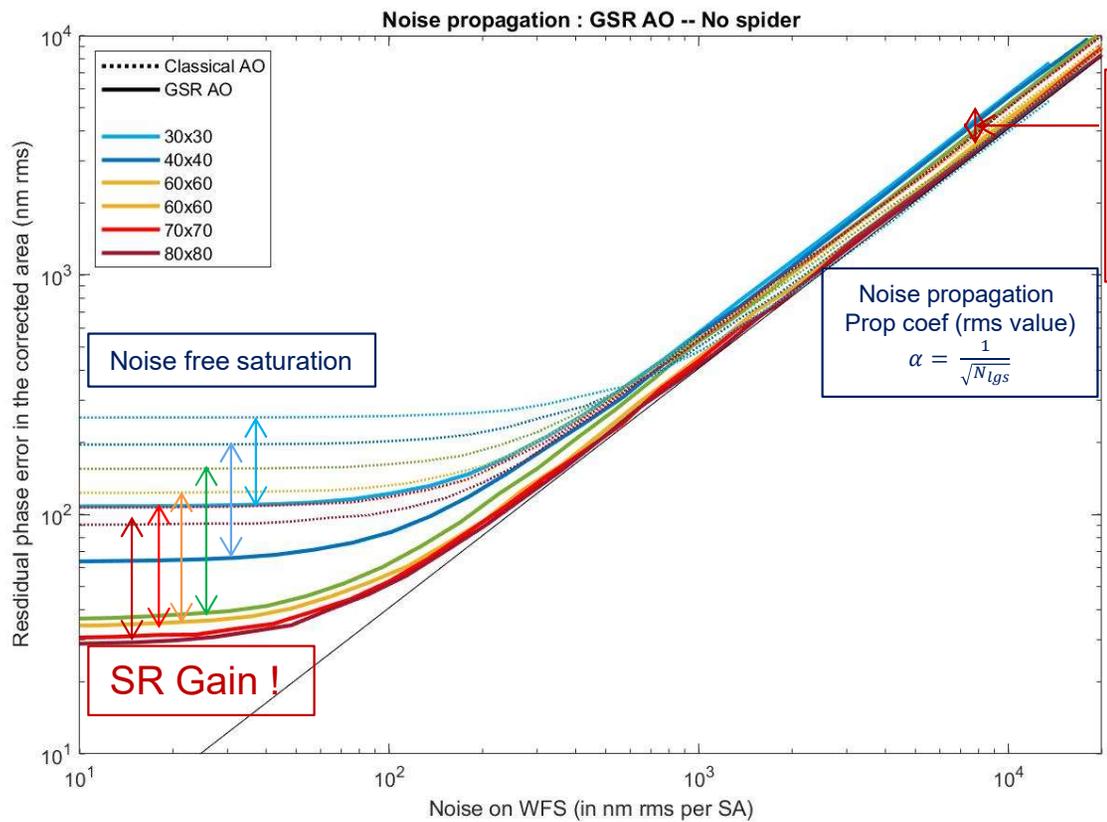


Small reduction of the noise propagation coeff (depends on the number of corrected modes)
 $\sim \ln(\# \text{ modes}) = \ln(\# \text{ SA})$ for class AO

Credit: Thierry Fusco

H-LTAO: Noise propagation for SR AO

M4 – IF Gauss
WFS Geo / 6 LGS
1 layer in the pupil

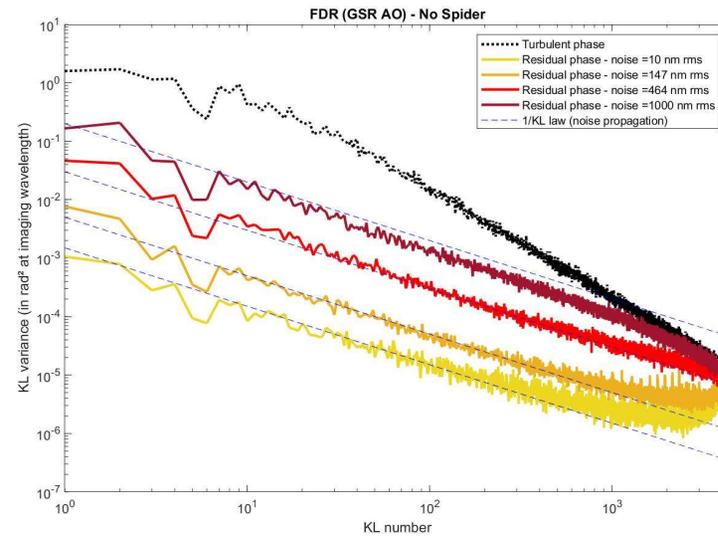
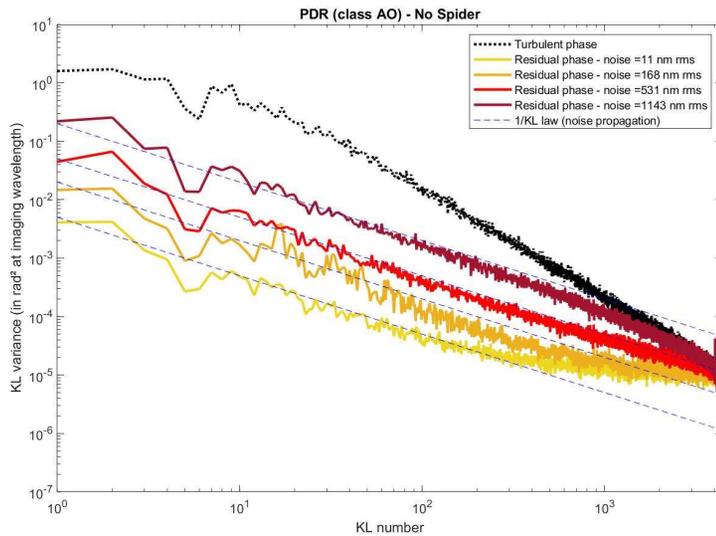


Credit: Thierry Fusco

SR gain is strong in the high SNR regime

H-LTAO: Noise propagation on PSD

Credit: Thierry Fusco



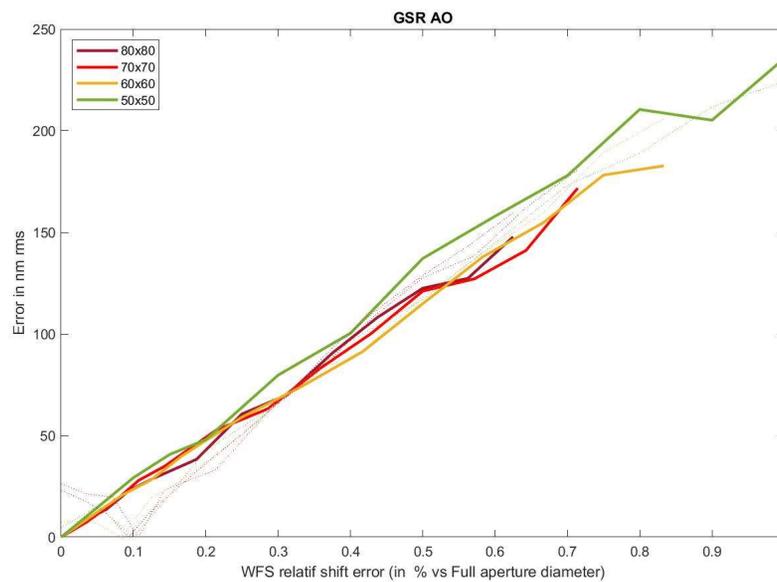
Same noise propagation behavior w/wo SR (1/k in variance)

■ Confirmation of previous results about the non degradation of noise propagation

H-LTAO: Impact of model errors

Difference between real registration and those introduced in IM model

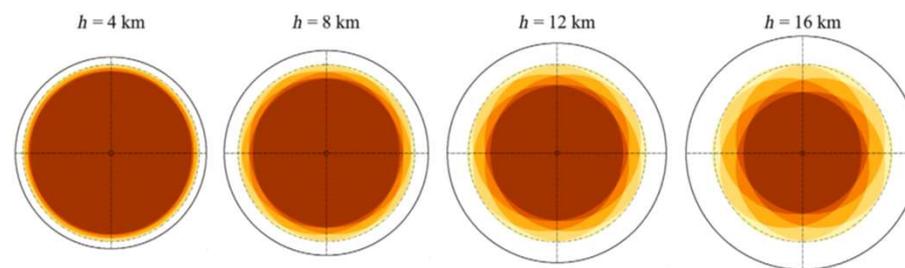
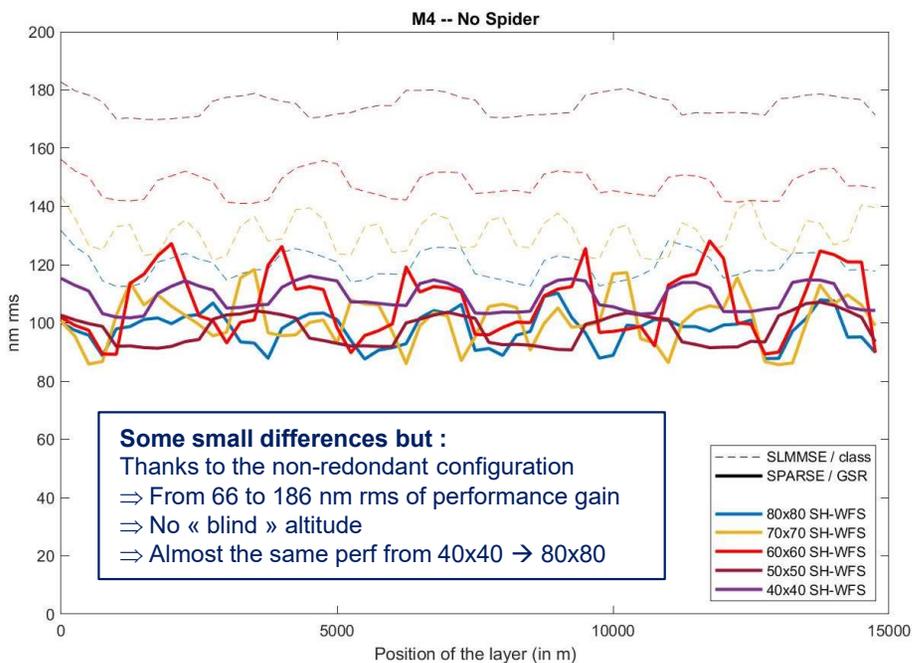
Credit: Thierry Fusco



- No significant difference between classical and SR cases
- Same error behavior hence same requirement !

H-LTAO: SR with full tomography

Credit: Thierry Fusco



	# SA	80x80	70x70	60x60	50x50	40x40
Average rms $\sqrt{\sum_0^{h_{max}} \{rms_i^2\}}$		$h_{max} = 15km$				
Classical case (nm rms)		120	131	148	175	211
GSR case (nm rms)		98	101	107	98	109

- SR comes for free for multi WFS systems when the turbulence is in altitude
- With full tomography, the number of SA plays a small part in the overall performance
- Nevertheless, the edges of the metapupils can't benefit from SR



Super-resolution wavefront reconstruction

■ Presentation of recent A&A paper

- Definition and history
- Mathematical formalism in 1D
- Application to Shack-Hartmann
- Sensitivity analysis
- Numerical application

Astronomy & Astrophysics manuscript no. aa43954-22
September 9, 2022

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Super-resolution wavefront reconstruction

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Received 4 May 2022 / Accepted 2 August 2022

■ On sky verification with AOF

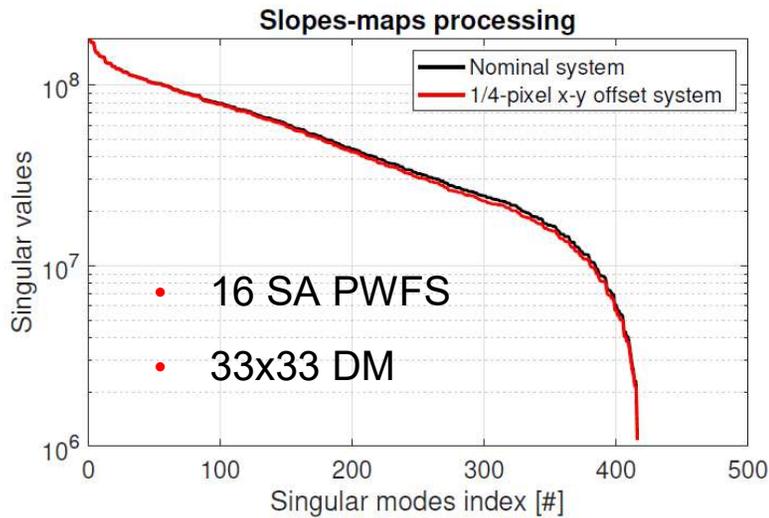
■ Application to future AO designs: MORFEO, MAVIS, H-LTAO, GMT ...

■ Super-resolution applied to the Pyramid WFS

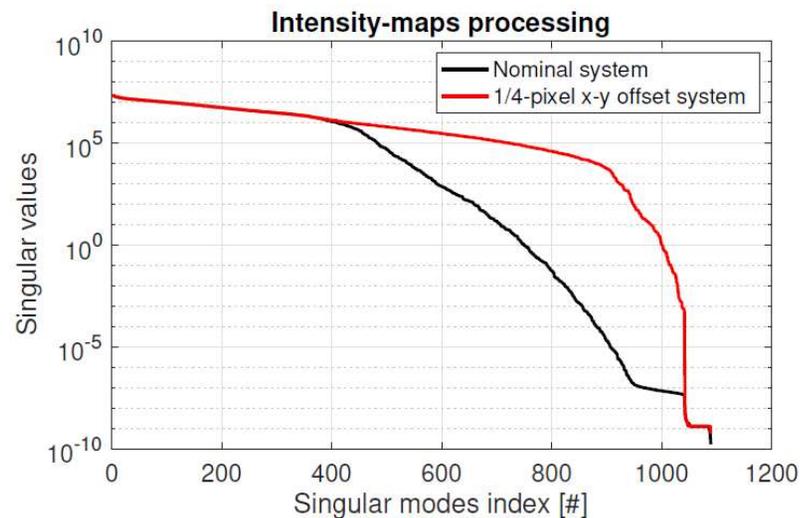
■ Next steps



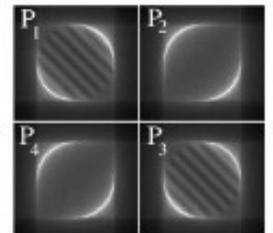
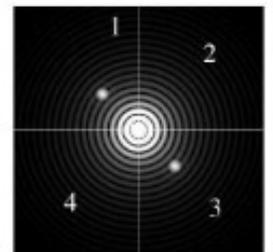
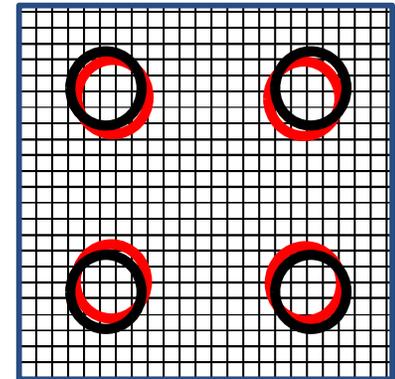
Super-resolution applied to a pyramid WFS



[Correia (2022)]



[Guyon (2005)]



- The PWFS is not exactly matching the SR formalism described before
- However, pairs of diagonally opposed quadrants see the same signal
- Preliminary simulations indicate that SR may be applied, IF pixels proc.

Future steps



- SR is baselined for most tomographic AO systems in design phase
- Technical time for additional on sky tests with AOF/GALACSI
- Evaluation of SR with the Pyramid WFS on the Ghost bench
- Learn from MAVIS, HARMONI and MORFEO during their MAIT phase
- Future possible applications:
 - MORFEO LOR (NGS truth loop) ?
 - ELT GLAO ?
 - Global MCAO ? [[Ragazzoni \(2013\)](#)]
 - Mono WFS approach with Taylor frozen flow [[Rihuan & Wagner \(2020\)](#)]
 - New bio-edge concept [[Verinaud \(2022\)](#)]

Thank you ! Any questions ?



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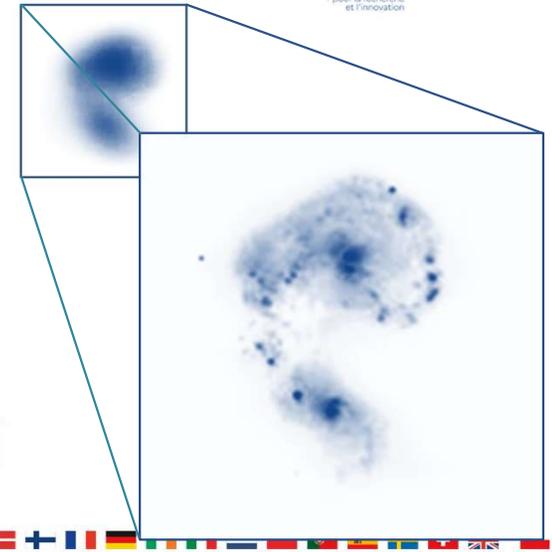


25 to 30th June 2023

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Dealiasing: Fourier space

