Bi-O-edge sensors

The Foucault knife-edge advantage in the race to sensitivity

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Outline

- Foucault Knife Edge (FKE) test by Léon Foucault (1859): one of the oldest sensor to measure optical surfaces

- Modification of 2 existing opto-mechanical Concepts enabling to use 2 ⊥ FKEs in a single stage (X)AO

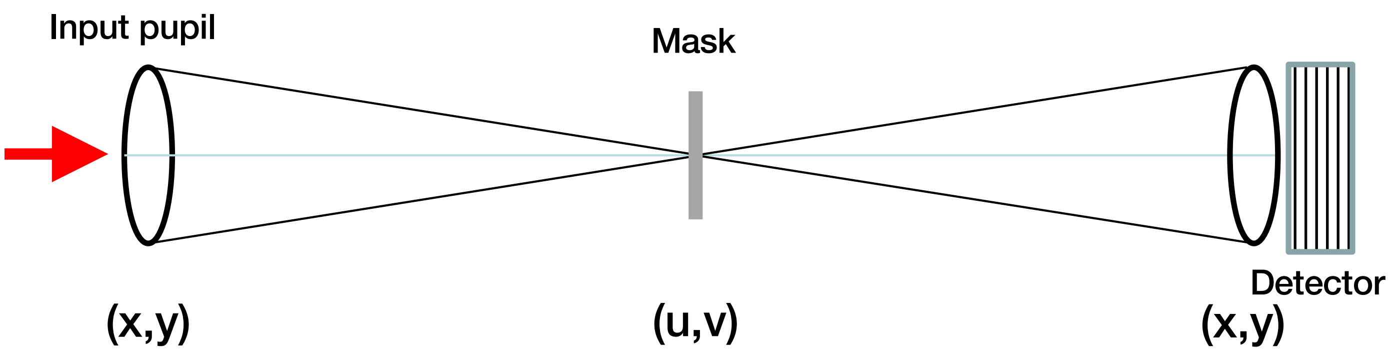
→ Bi-Orthogonal-Foucault-knife-edge sensors, in short

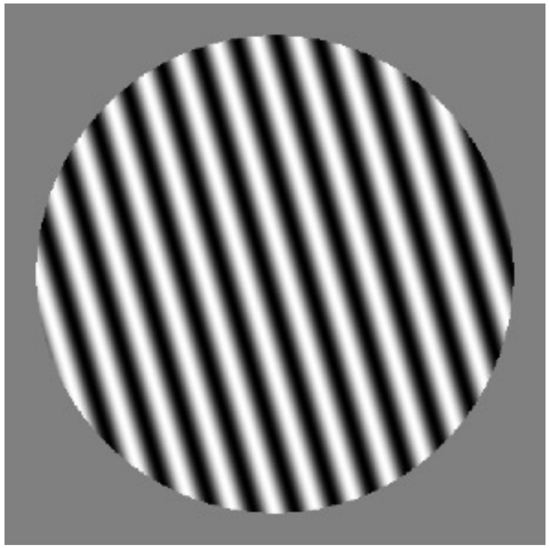
Bi-O-edge

- Noise propagation analysis :
  - Visual description: single Fourier mode analysis (O. Guyon 2005)
  - Convolutional model (C-model): O. Fauvarque, V. Chambouleyron
  - End-to-End simulations (OOPAO, T. Héritier)

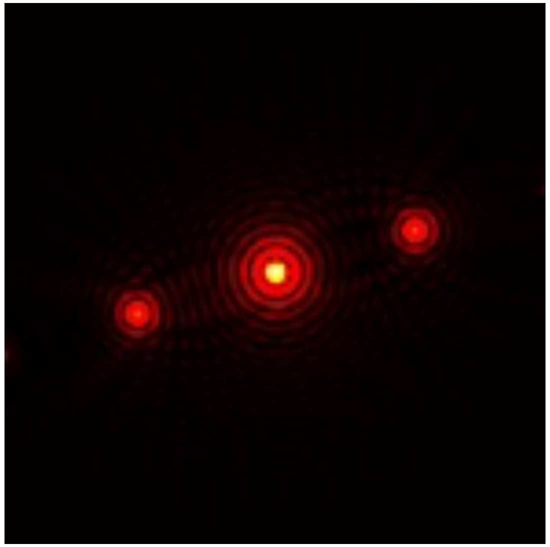
- Super-Resolution (Oberti 2022) the aliasing killer

The FKE:
A Fourier Filtering sensor

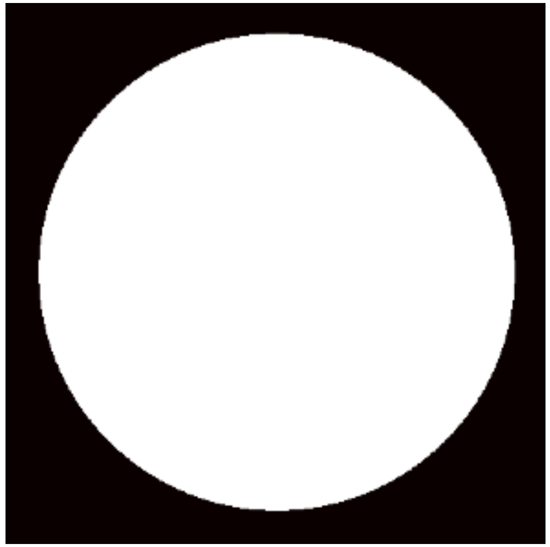




Wave-Front

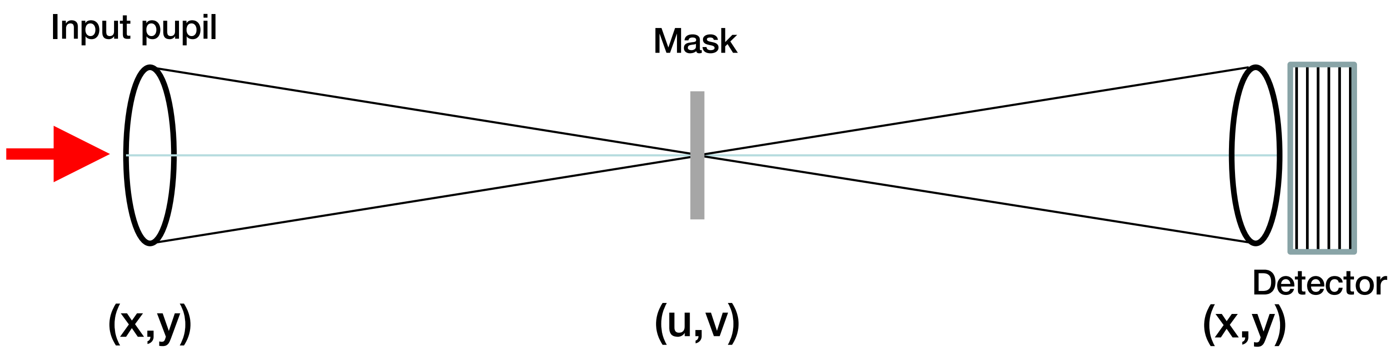


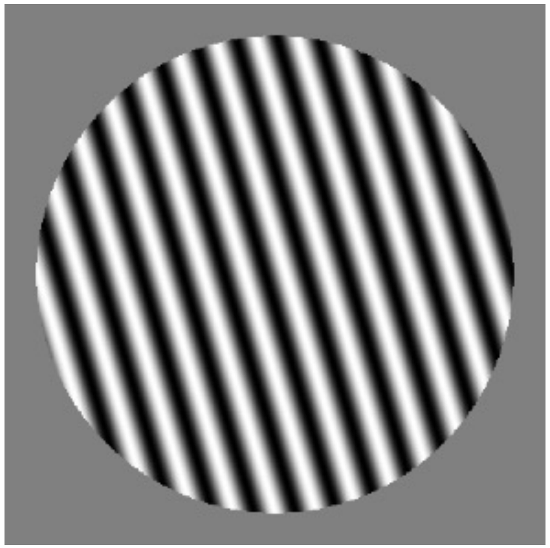
PSF



Pupil Intensity

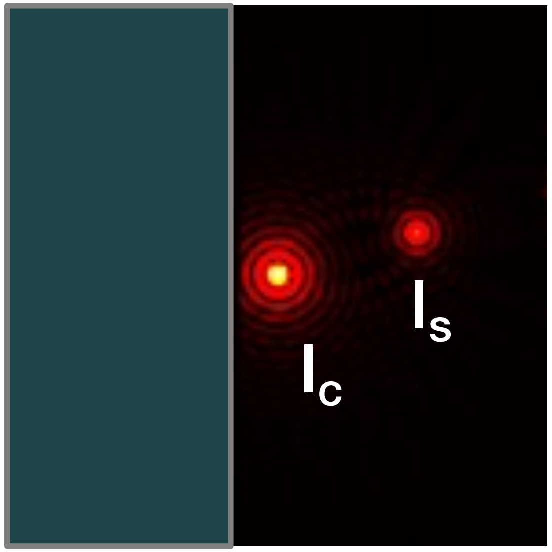
The FKE:
A Fourier Filtering sensor





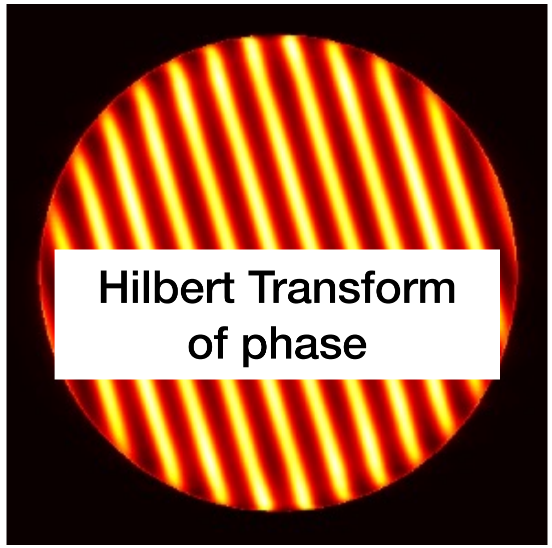
Wave-Front

STD(WF) = σ



PSF

$$\text{Fringes} = 2\sqrt{I_c I_s}$$



Pupil Intensity

STD (I(σ)-I(0)) = σ

2D Nature of measurements
(~valid for Pyramid and Bi-O-edge)

(x,y) → (u,v) Fourier space

S_x , S_y: slope-like Signal

LO: Low order, HO: High Order

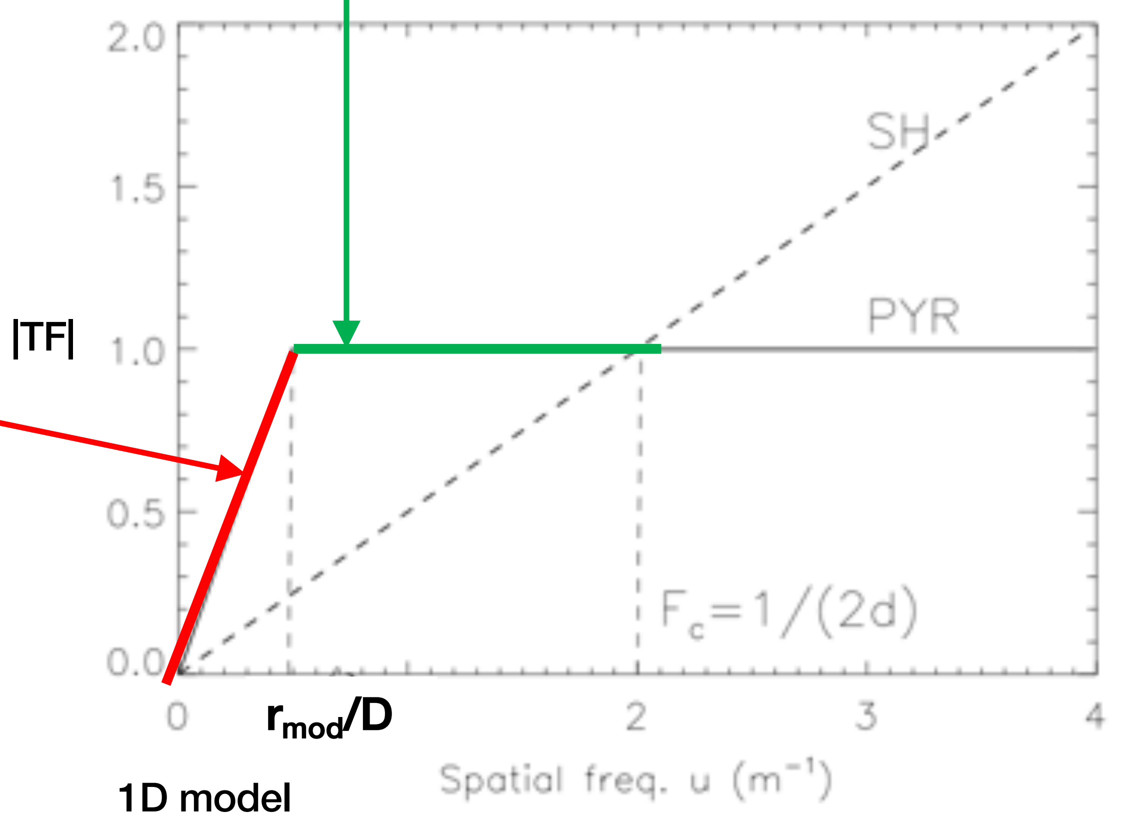
C-model→ 2D Transfer Functions

$$S_x[\phi_{HO}(x,y) = \mathbf{sgn}(\boldsymbol{u})\, i\ \hat{\phi}_{LO}(u,v)$$
$$= \pm$$
$$S_y[\phi_{HO}(x,y)] = \mathbf{sgn}(\boldsymbol{v})\, i\ \hat{\phi}_{LO}(u,v)$$

Each component contains all and same info(Hilbert T)

$$S_x[\phi_{LO}(x,y)] = i\, \boldsymbol{u}\, \hat{\phi}_{LO}(u,v)$$
$$\neq$$
$$S_y[\phi_{LO}(x,y)] = i\, \boldsymbol{v}\, \hat{\phi}_{LO}(u,v)$$

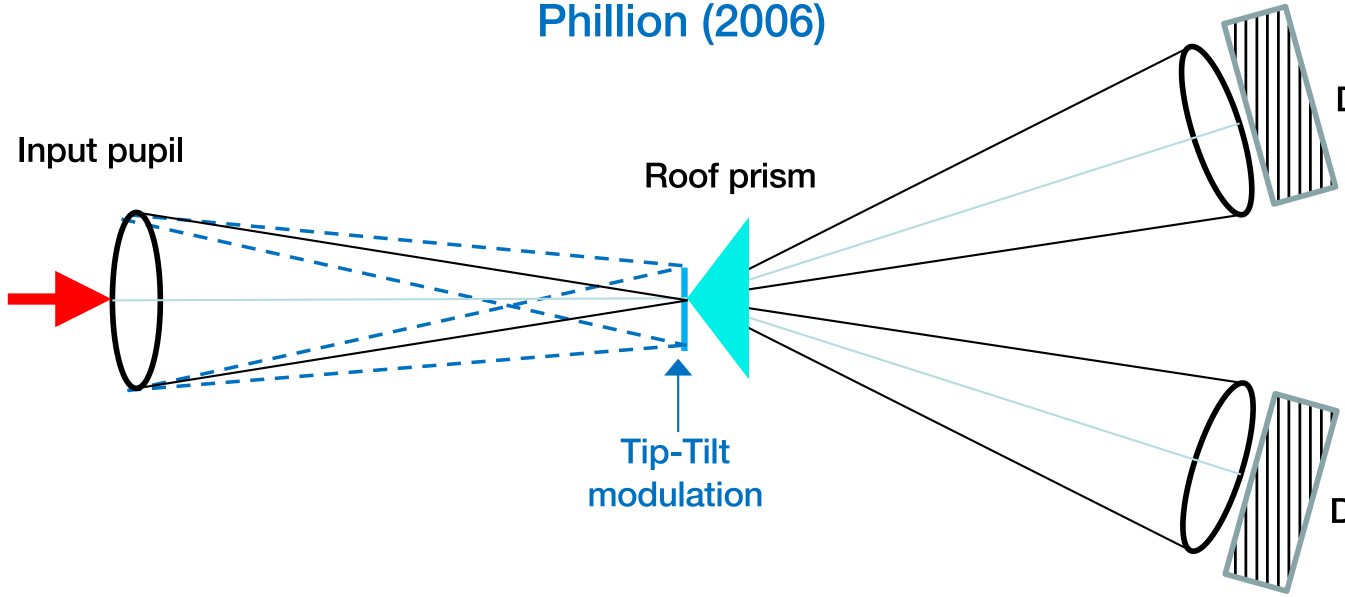
Each component contains half of the information. (slopes x,y)



1D model
Verinaud (2004)

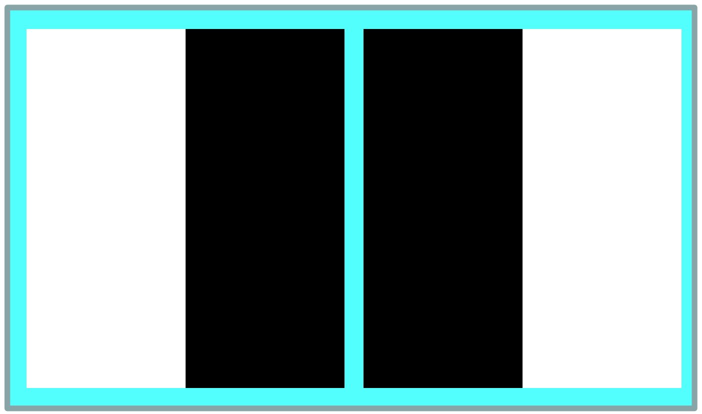
Loss-less FKE: refractive version

Phillion (2006)



FKE: Binary amplitude mask with <u>sharp edges</u>

Mask 1 Mask 2



White = 1: light goes through
Black = 0: light is blocked

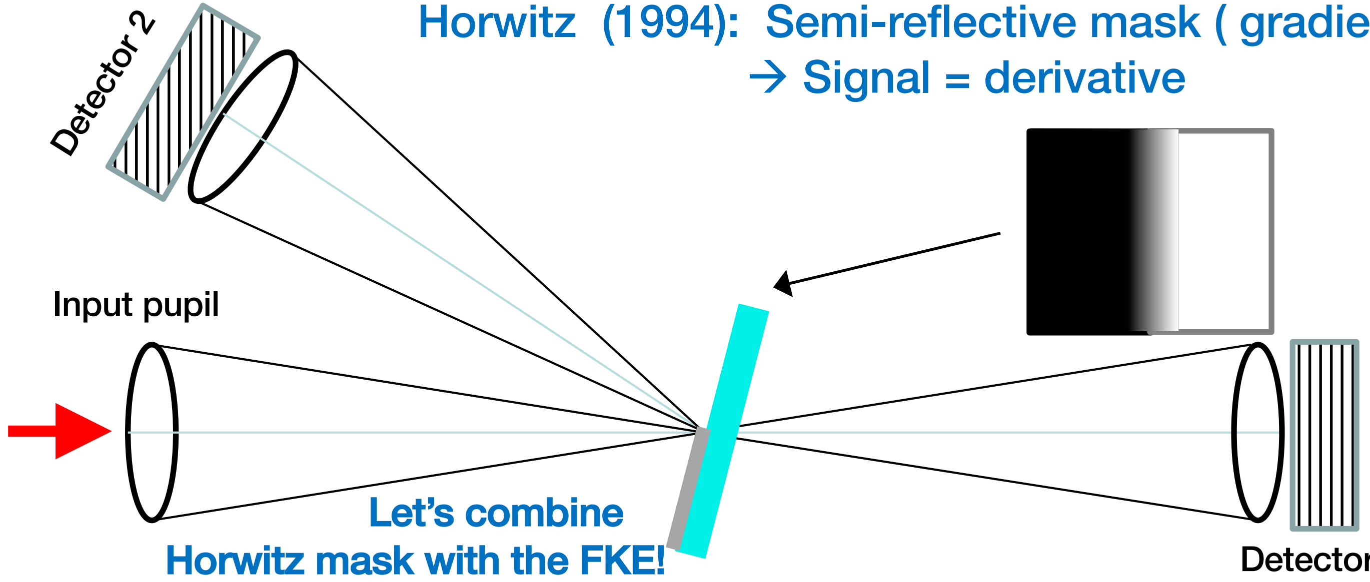
Beam modulation → Signal = derivative
(Ragazzoni, 1996)

Valid only for | f | < r_{mod}/D_{tel} (geom. model)

D_{tel} = telescope diameter
f^2=u^+v^2: spatial frequencies
r_{mod} = modulation radius [λ / D]

Loss-less FKE: reflective version

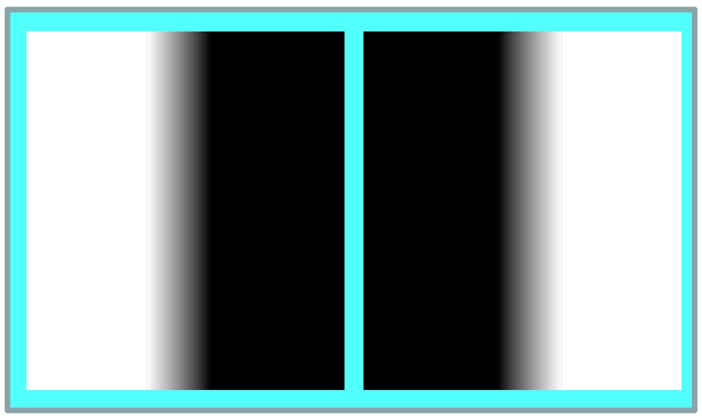
Horwitz (1994): Semi-reflective mask (gradient refl.)
→ Signal = derivative



FKE+ Grey edge of width $2\sqrt{2}\, r_{mod}$ (~ 100 microns)

→ Same range than the one given by dynamic modulation

Mask 1 Mask 2



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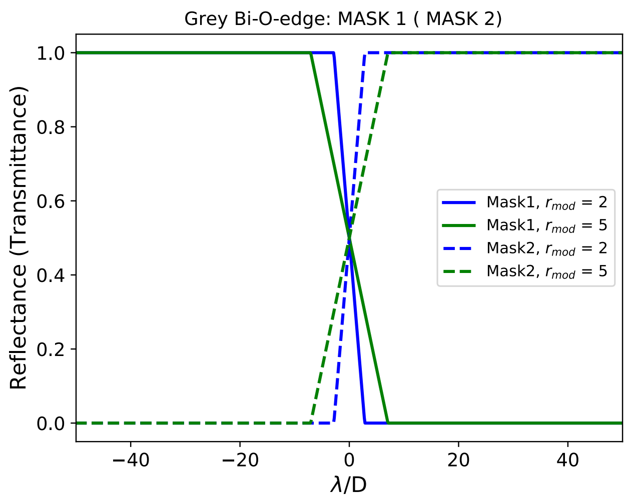
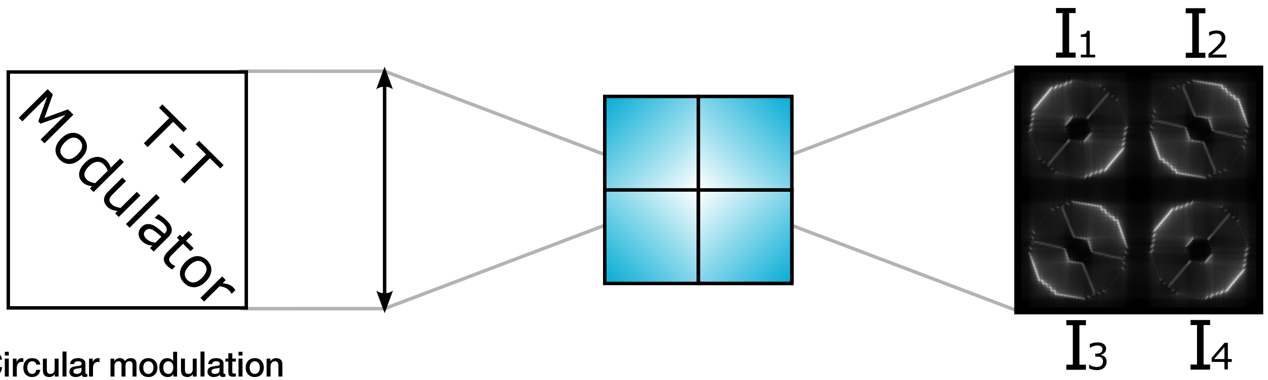


Let’s do a Match between 3 concepts

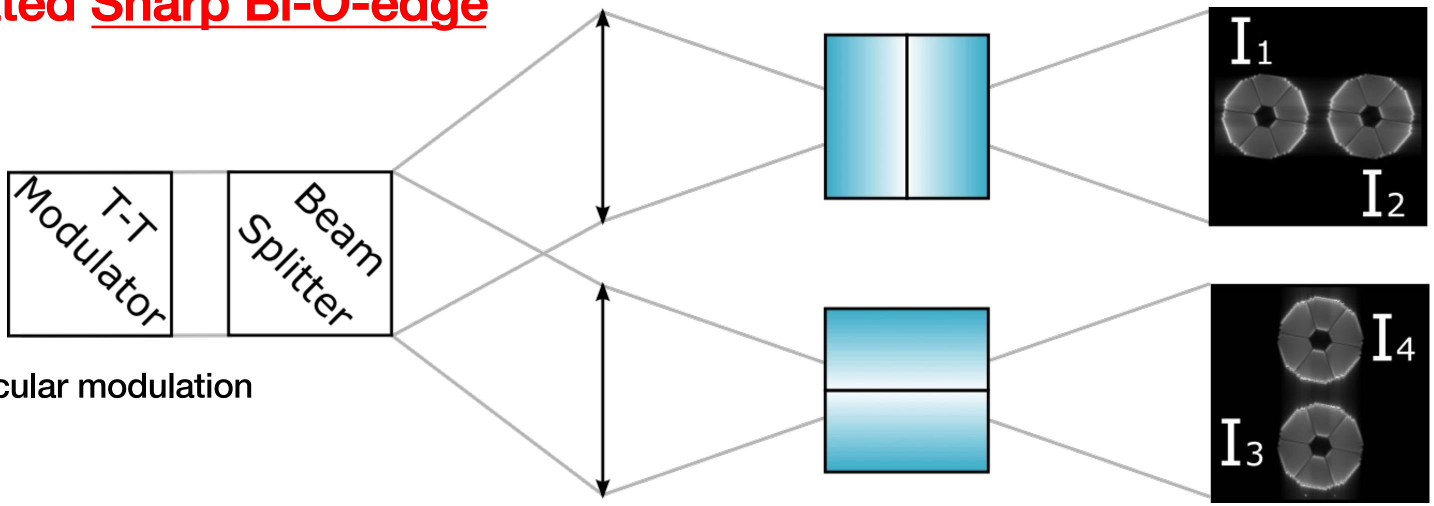
Refractive concepts

Modulated Pyramid



Circular modulation

Modulated Sharp Bi-O-edge

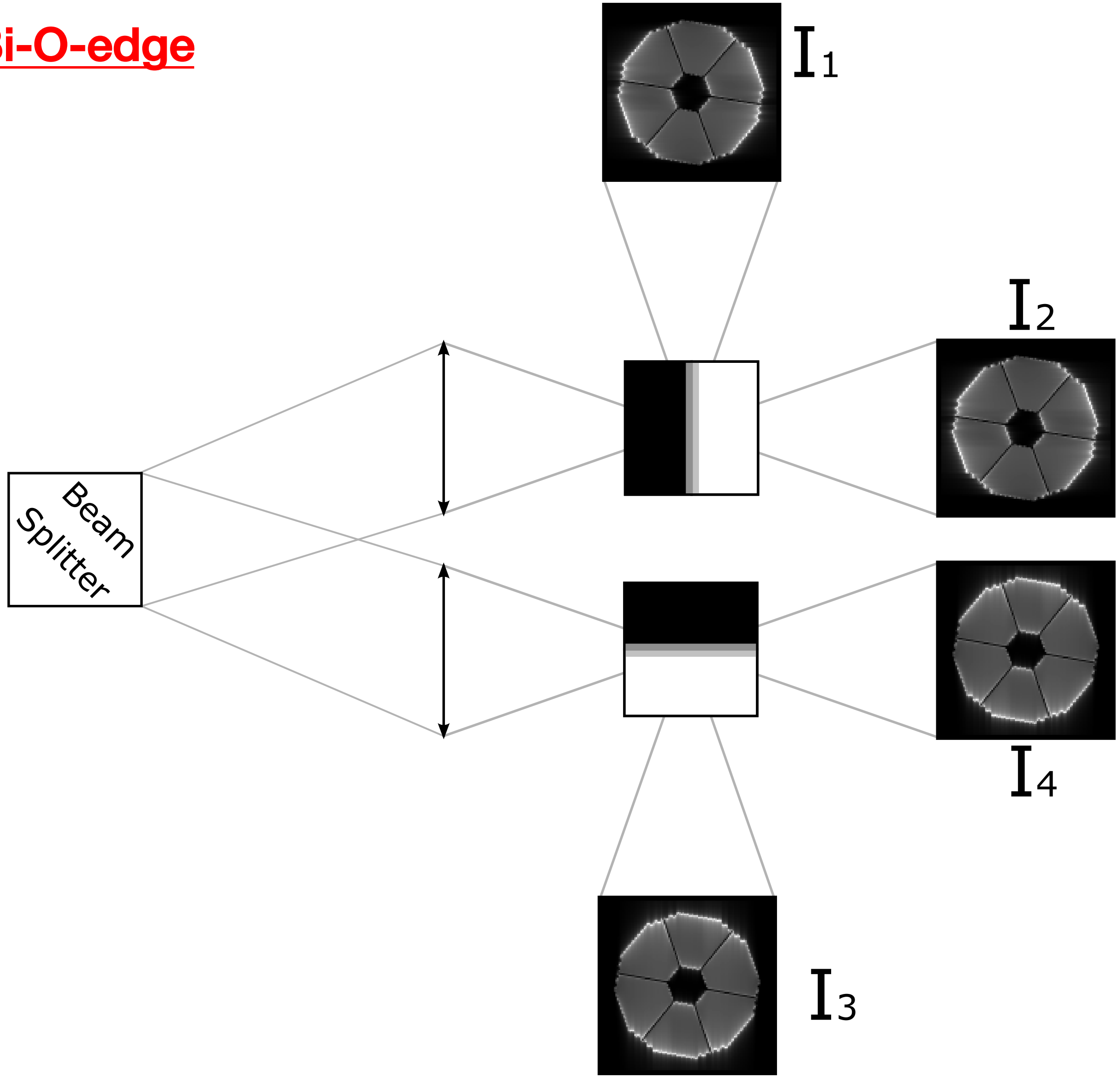


Circular modulation

Reflective concept

Grey Bi-O-edge

****
MODULATOR
GONE!
****



Sensitivity and photon noise propagation

$$\sigma^2_{error} = \sigma^2_{fitting} + \sigma^2_{tempo} + \sigma^2_{alias} + \boldsymbol{\sigma^2_{ph}}$$

- σ^2_{ph}: Photon noise propagated by WF reconstruction

$$\boldsymbol{\sigma^2_{ph}(u,v) = \frac{1}{|SEN(u,v)|^2}\sigma^2_N}$$

(Fauvarque 2016)

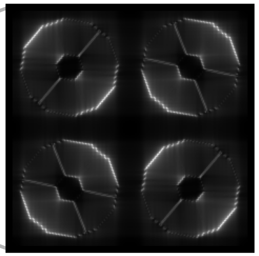
- Signal = user-defined meta-Intensity (Slope-like, Full-pixel)
- SEN = Sensitivity = STD (Signal) in response to input phase
  - Related to Fringe amplitude approximation
  - Transfer Function for C-model
- $\boldsymbol{\sigma^2_N}$: VAR (Signal(0)) due to photon noise only
- N : mean incident photons per sub-aperture, per frame, RON = 0 (averaged out over the pupil)

Definition of signal and computation of noise on signal

- <u>Slope-like signal</u>: definition such that Pyramid and Bi-O-edge give <u>same signal in geometrical approximation</u>
- Poissonian noise (k=1 to 4): $VAR(I_k(x,y)) = MEAN(I_k(x,y)) = N/4$

PYRAMID

I_1 I_2

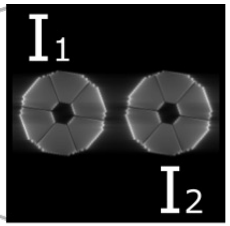


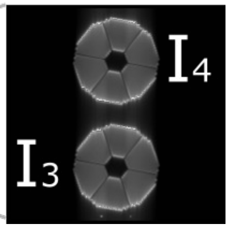
I_3 I_4

$$S_{X,pyr} = \frac{I_2 + I_4 - I_1 - I_3}{N} \qquad S_{Y,pyr} = \frac{I_1 + I_2 - I_3 - I_4}{N}$$

$$\boldsymbol{\sigma^2_{N,pyr} = \frac{1}{N^2}\left(\frac{4N}{4}\right) = \frac{1}{N}}$$

BI-O-EDGE





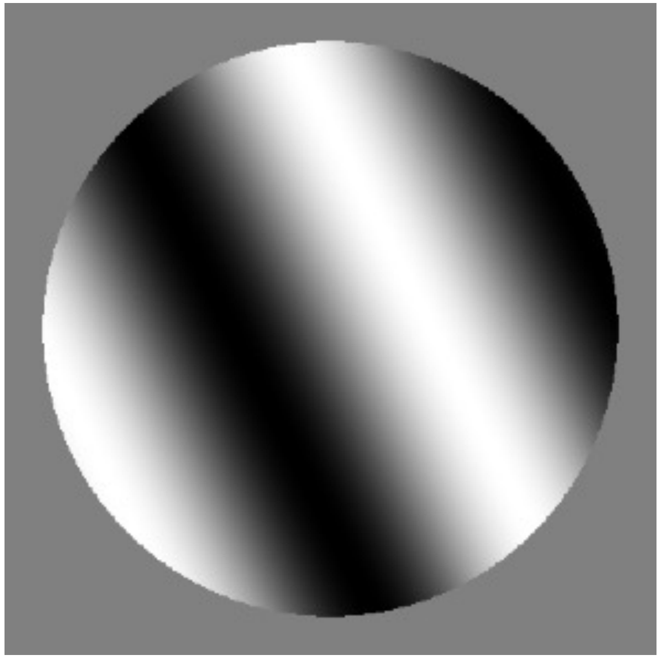
$$S_{X,bio} = \frac{I_2 - I_1}{N/2} \qquad S_{Y,bio} = \frac{I_4 - I_3}{N/2}$$

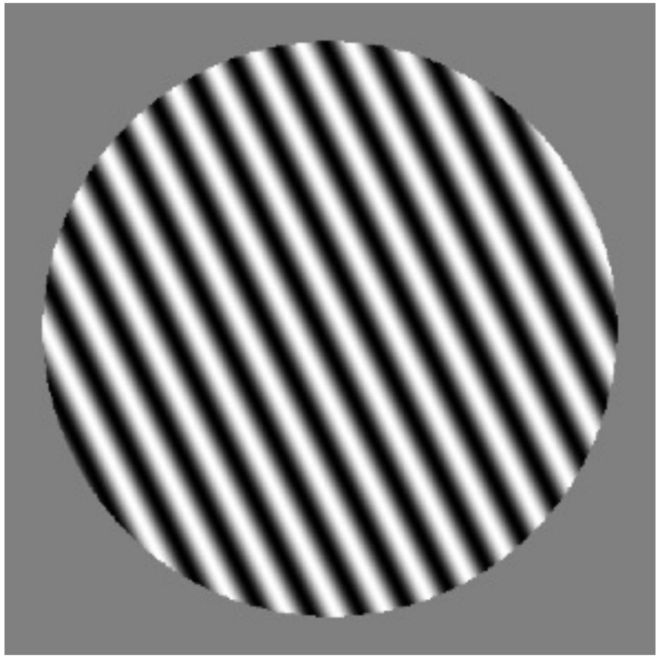
$$\boldsymbol{\sigma^2_{N,bio} = \frac{1}{(N/2)^2}\left(\frac{2N}{4}\right) = \frac{2}{N}}$$

Noise on signal is twice larger than for Pyramid

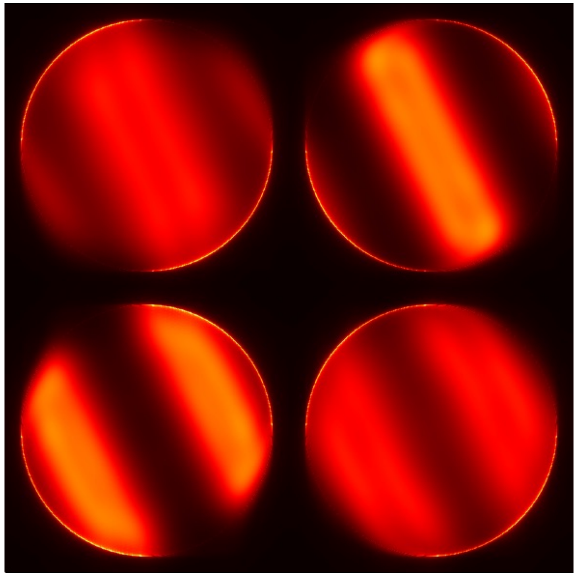
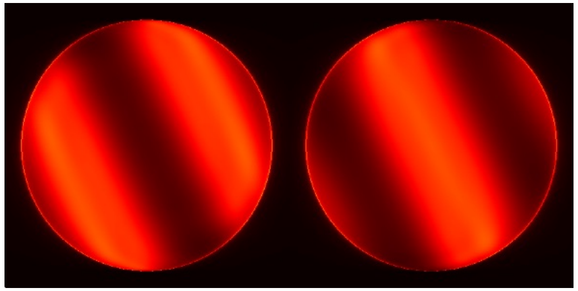
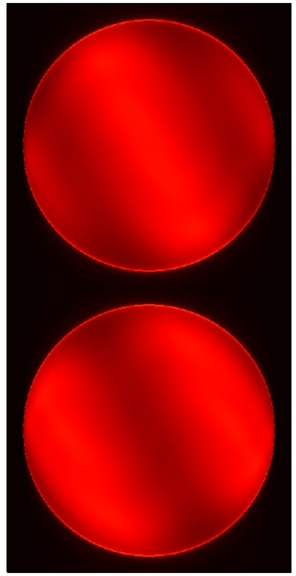
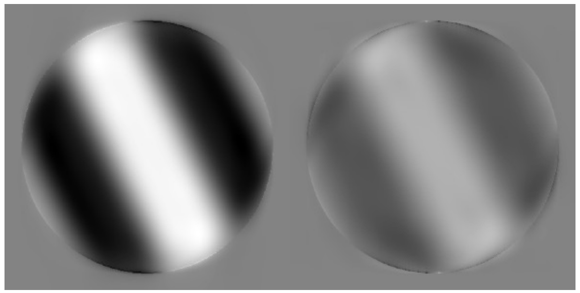
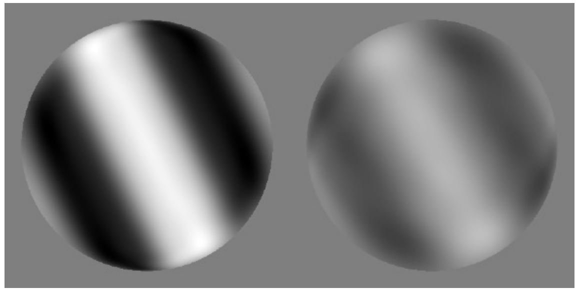
Pure Fourier modes as Test Wave-Fronts

- Diffraction by sharp edges are ignored (unless specified)
- D_{tel}: Telescope diameter. r_{mod}: modulation angle
- LO: $|f| < r_{mod} / D_{tel}$
- HO: $|f| > r_{mod} / D_{tel}$





LO modes: PYR vs BIO (sharp and grey)

	Pyramid	Bi-O-edge	
Intensities			
Slope-like Signal			

LO modes: PYR vs BIO (sharp and grey)

- All 3 sensors have the same slope-like sensitivity, but the Bi-O-edge signal is twice more noisy → Noise propagated is twice larger
- →The Bi-O-edge needs twice more photons than the Pyramid for reaching the same precision to sense and correct LO modes

$$SEN_{pyr,LO} = SEN_{bio,LO} = SEN_{LO}$$

$$\sigma^2_{ph,pyr,LO} = \frac{1}{SEN^2_{LO}}\sigma^2_{N,pyr} = \frac{1}{SEN^2_{LO}}\frac{1}{N}$$

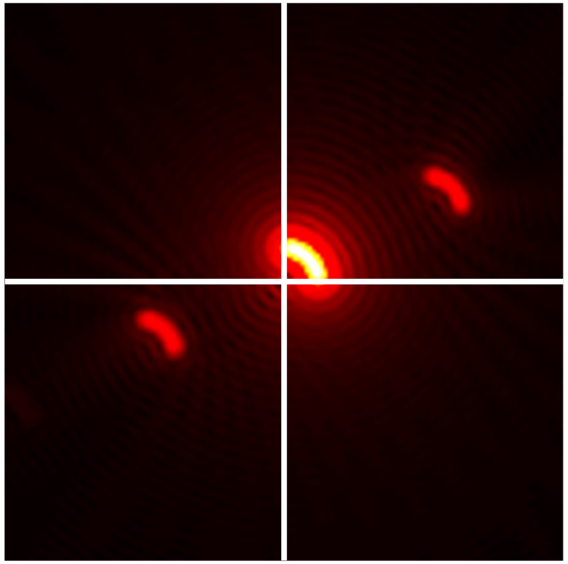
$$\sigma^2_{ph,bio,LO} = \frac{1}{SEN^2_{LO}}\sigma^2_{N,bio} = \frac{1}{SEN^2_{LO}}\frac{2}{N}$$

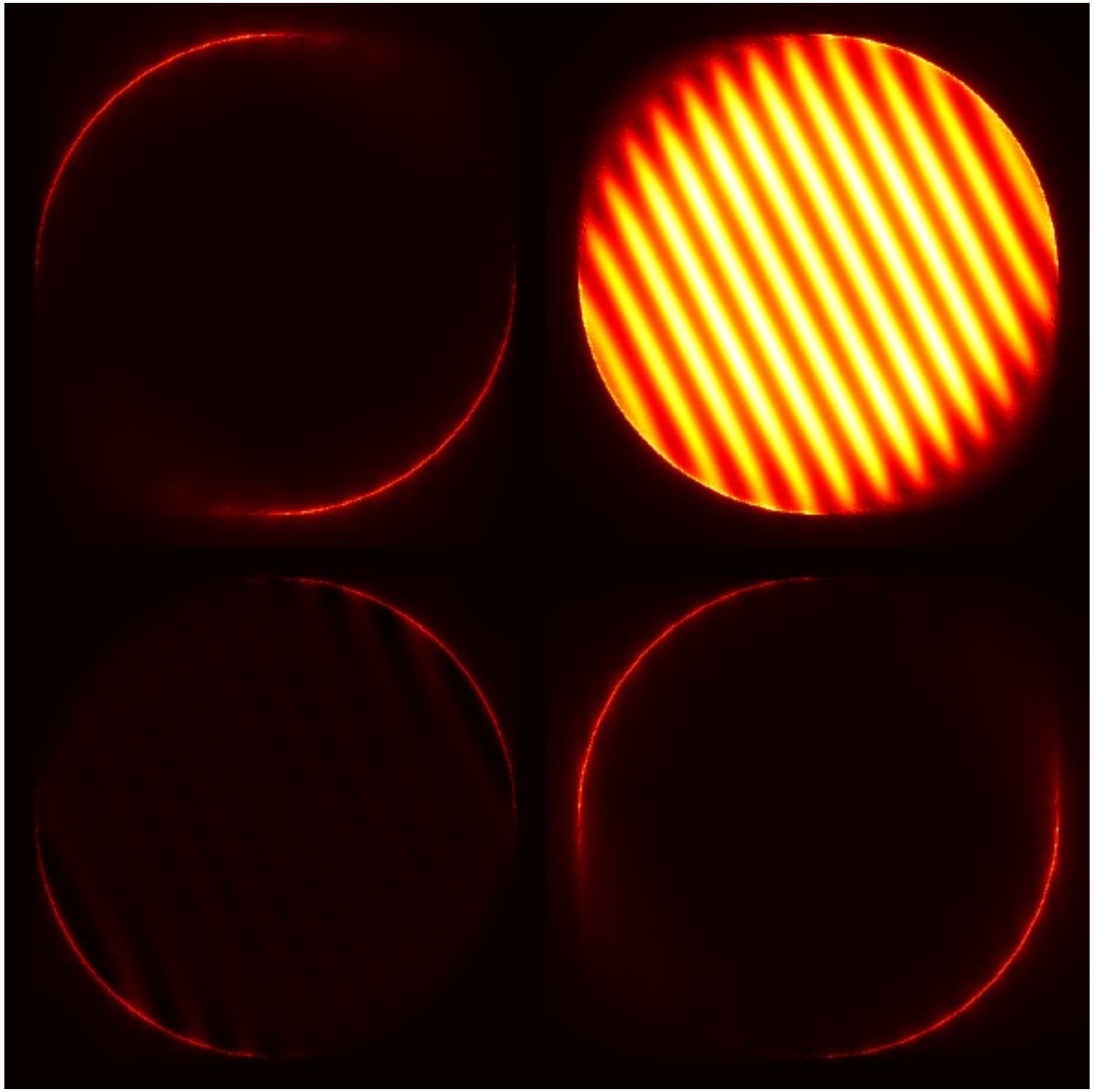
→

$$\boxed{\sigma^2_{ph,bio,LO} = 2\,\sigma^2_{ph,pyr,LO}}$$

HO modes: PYRAMID

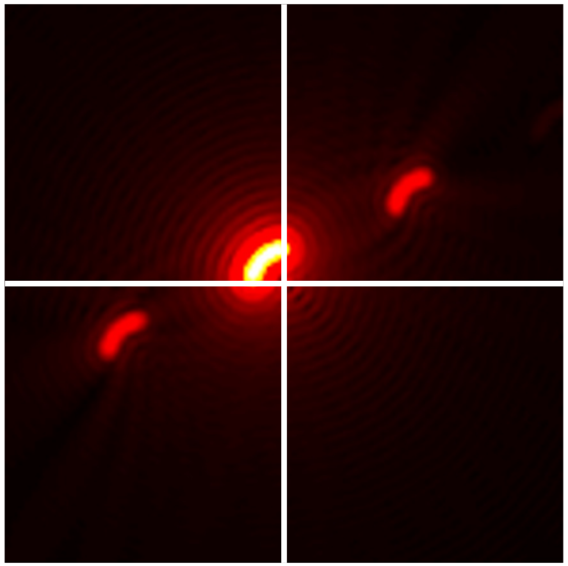
Pyramid edges

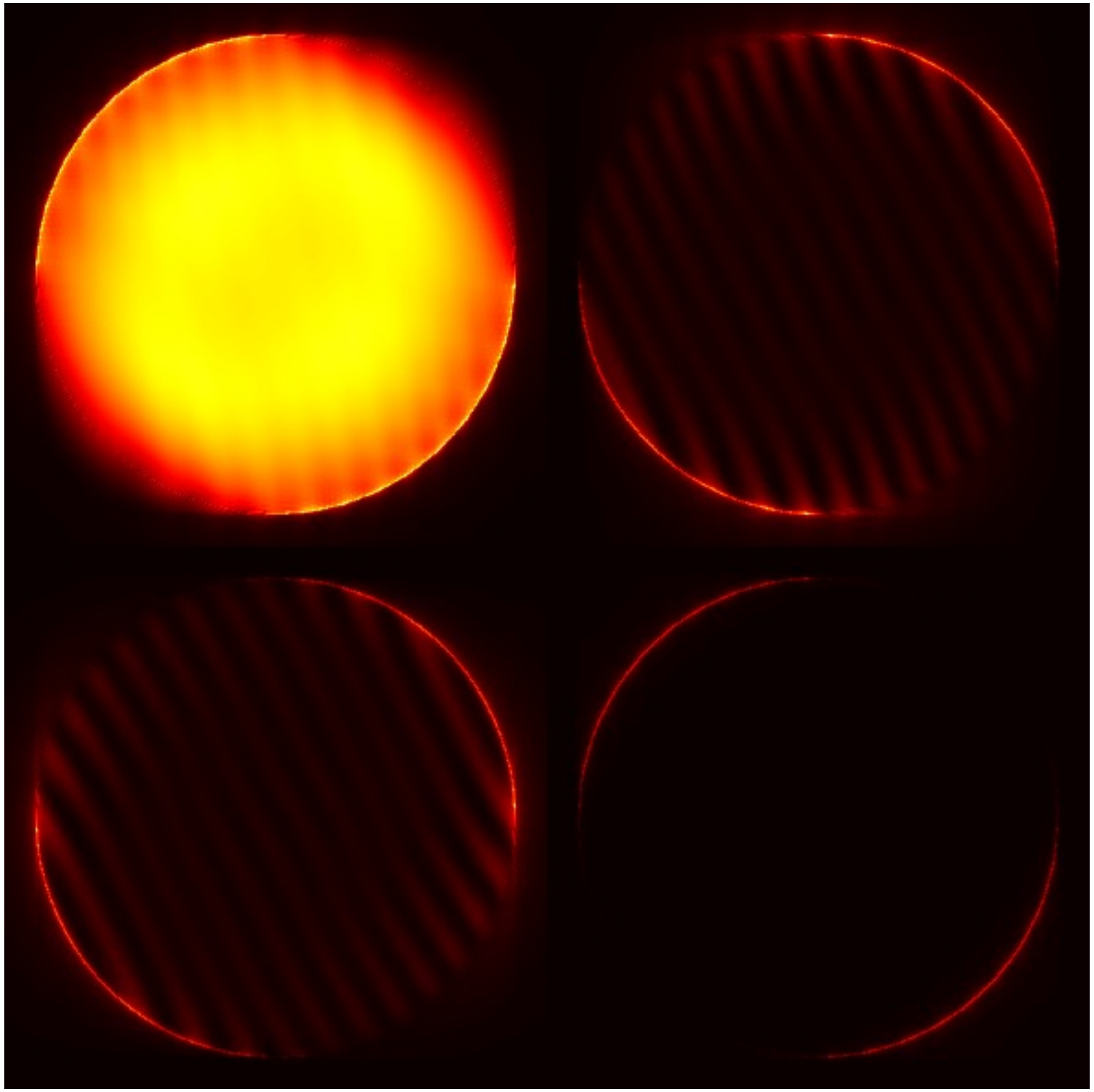




HO modes: PYRAMID

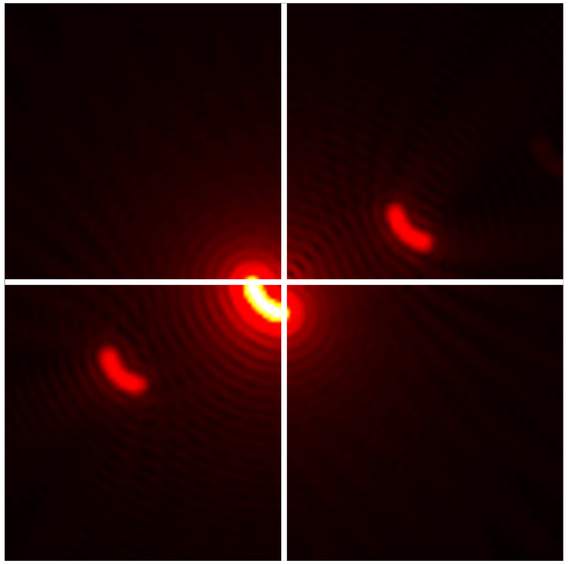
Pyramid edges

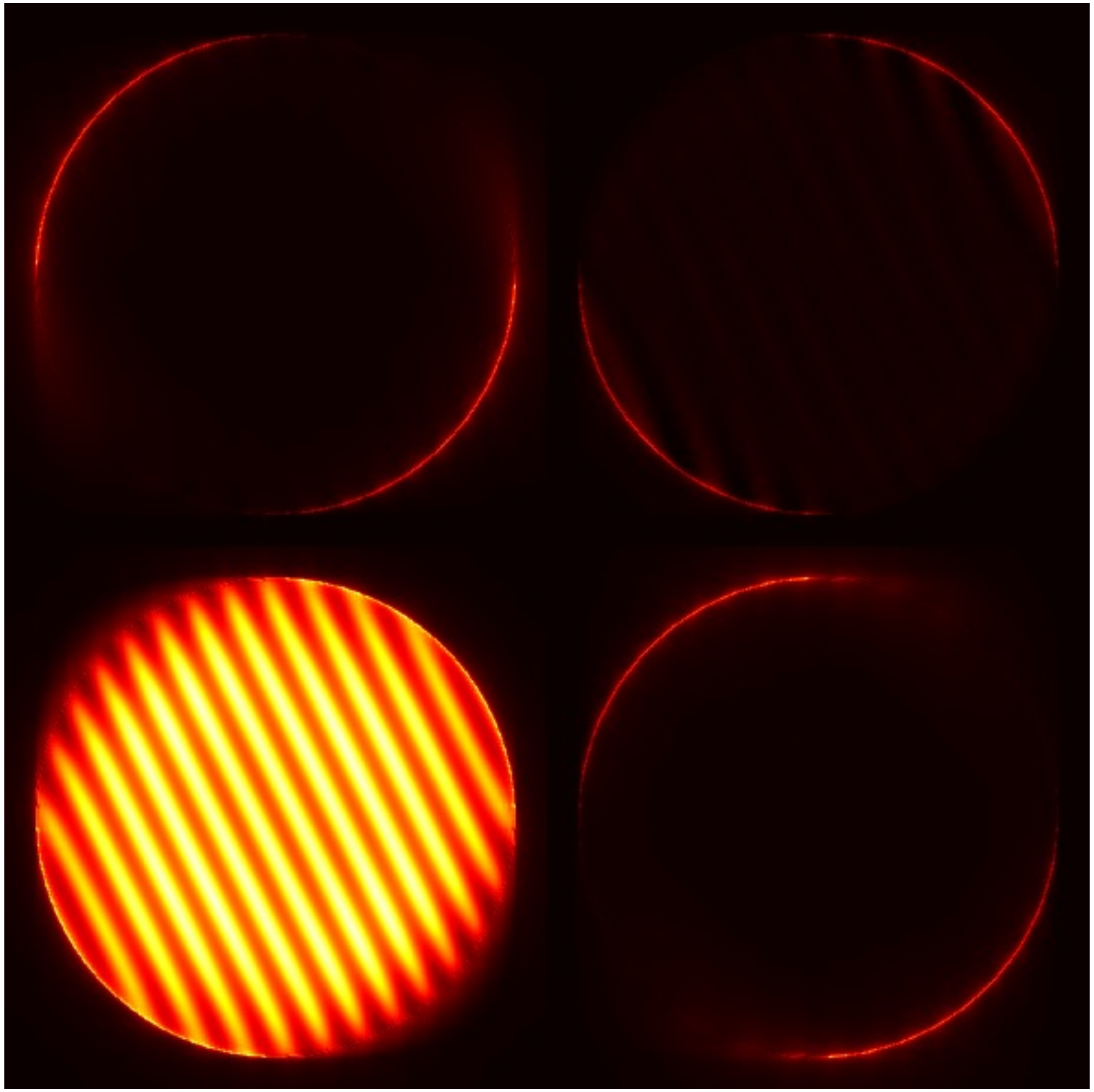




HO modes: PYRAMID

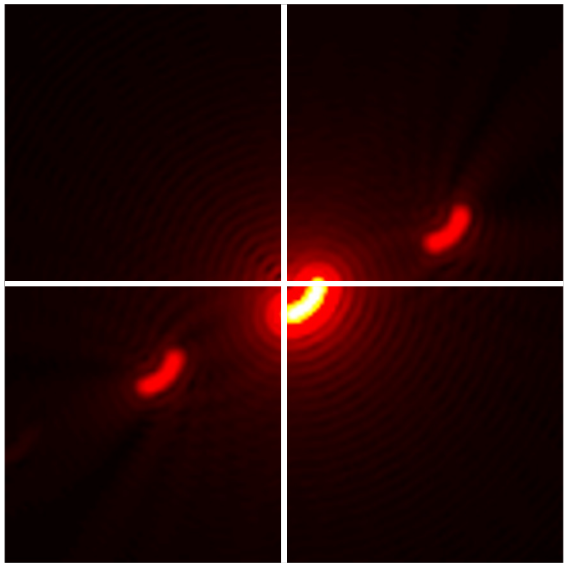
Pyramid edges

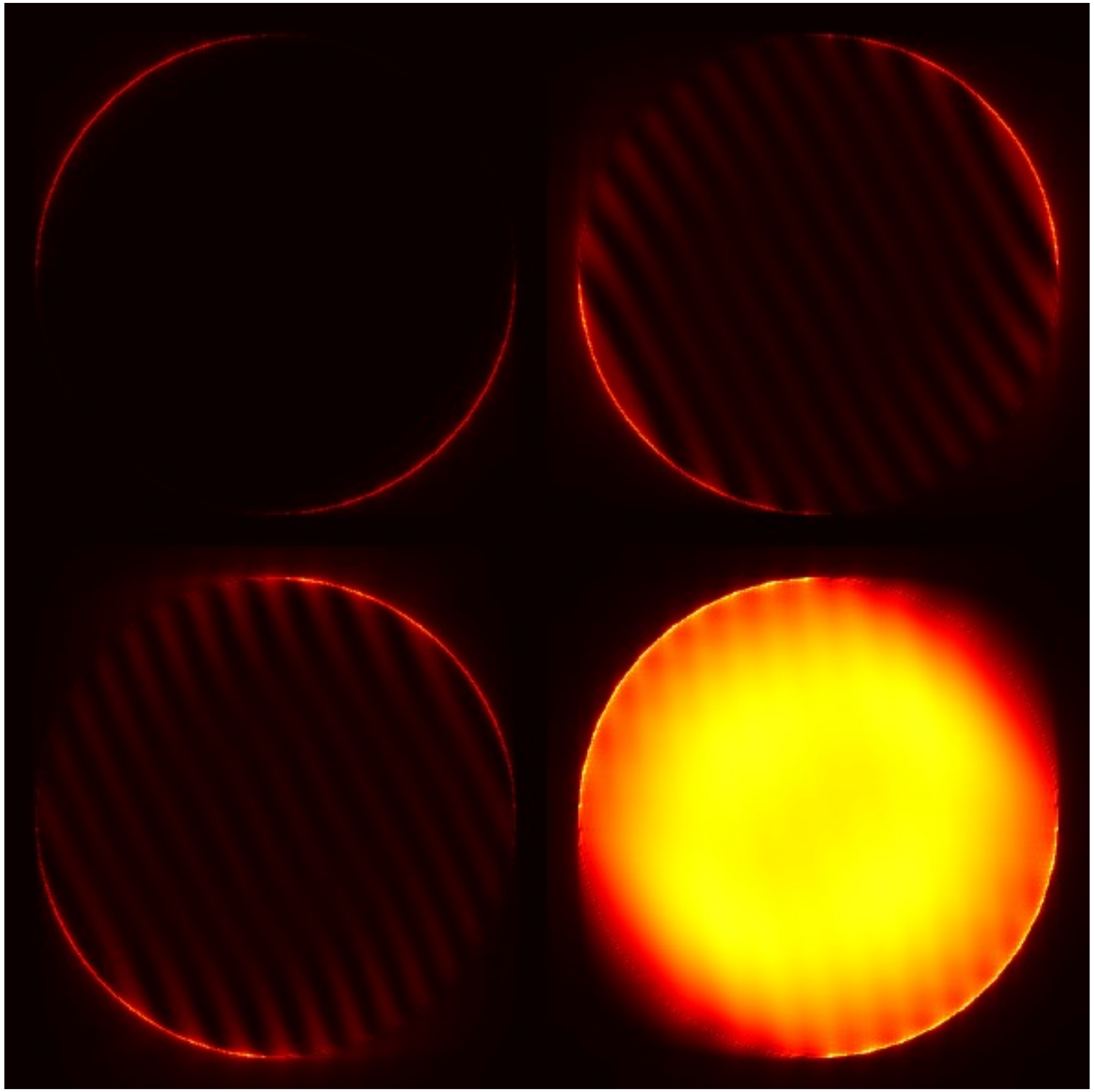




HO modes: PYRAMID

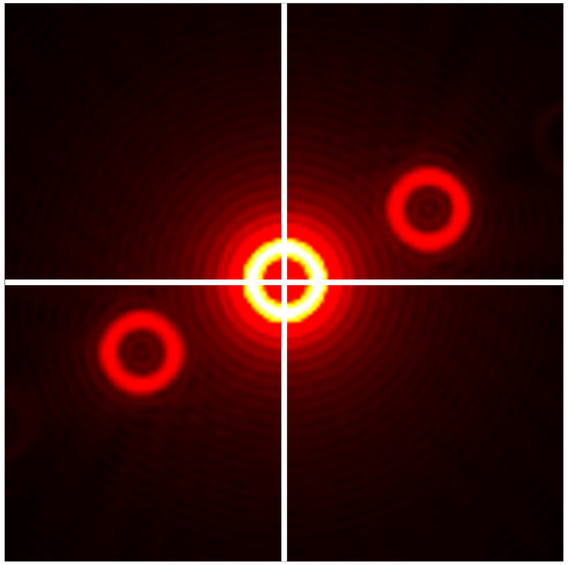
Pyramid edges

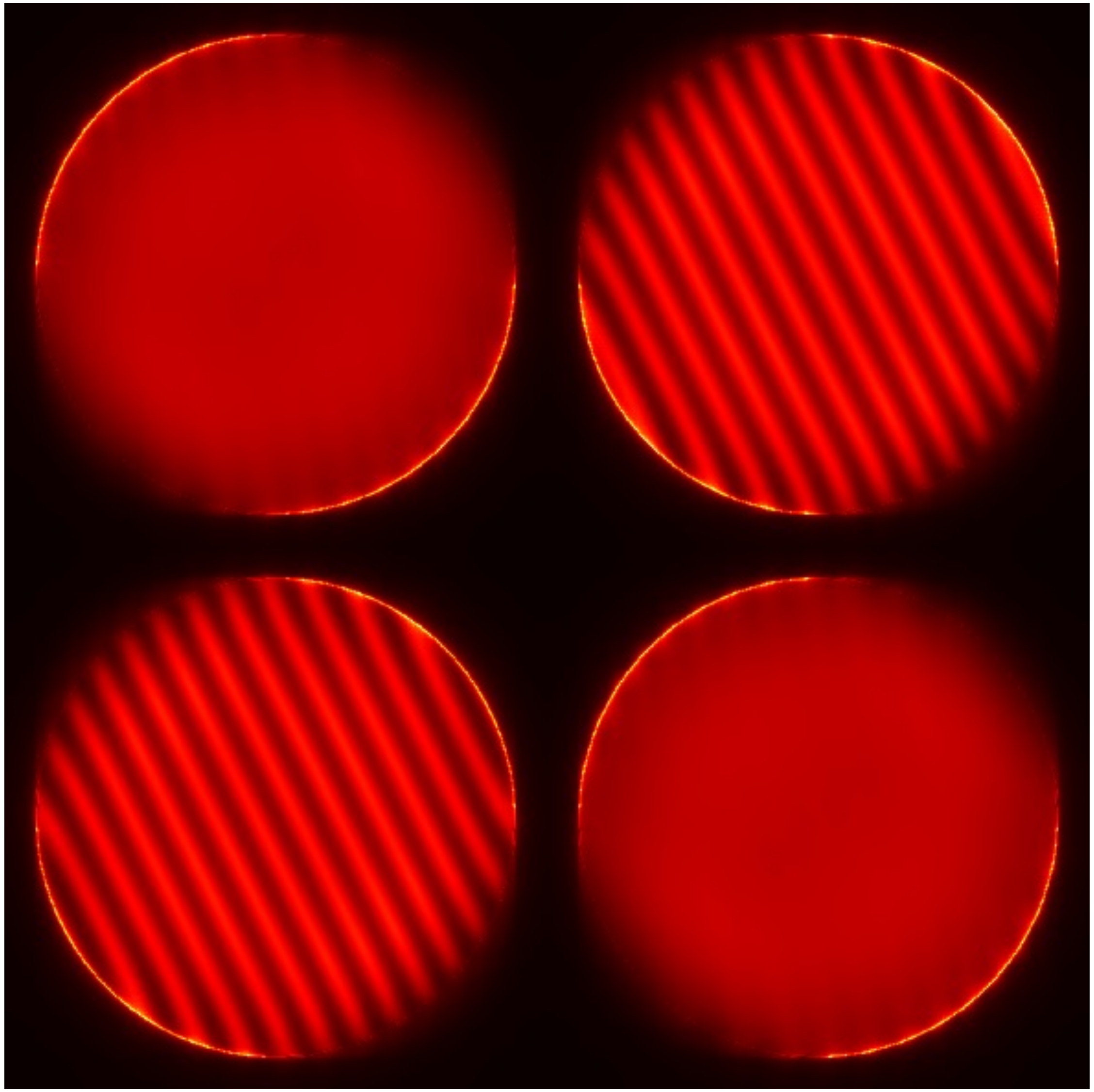




HO modes: PYRAMID

Pyramid edges





HO modes: PYRAMID

- Spatial Blindness per quadrant: 50%
- Modulation Duty Cycle per quadrant: 25%

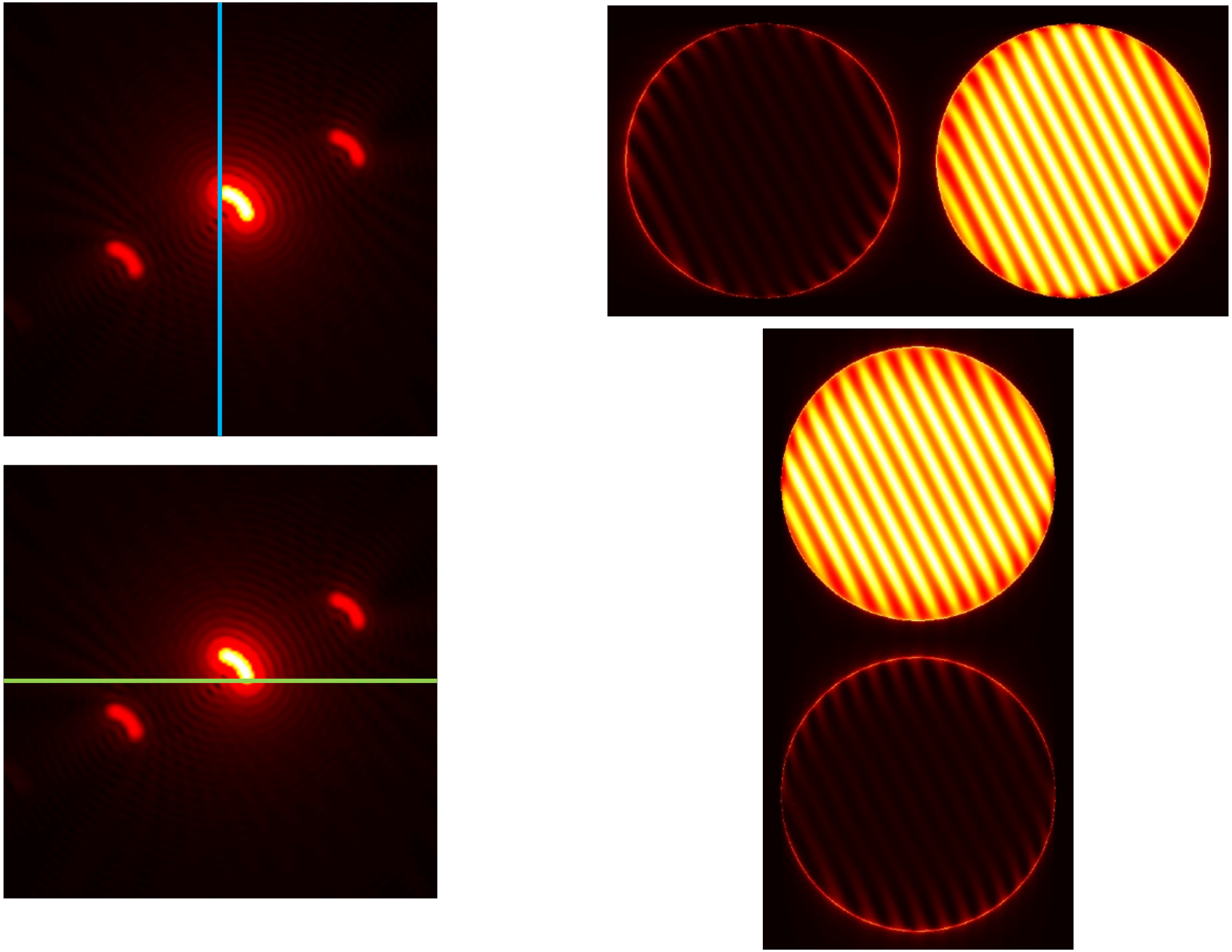
- → SENx = SENy = 2x0.25 = 0.5 → SEN^2=0.5

(diffraction by edges ignored)

- Noise propagated on HO modes is:

$$\sigma^2_{ph,pyr,HO} = \frac{1}{SEN^2_{pyr,HO}} \sigma^2_{N,pyr} = \frac{1}{1/2} \frac{1}{N} \longrightarrow \boxed{\sigma^2_{ph,pyr,HO} = \frac{2}{N}}$$

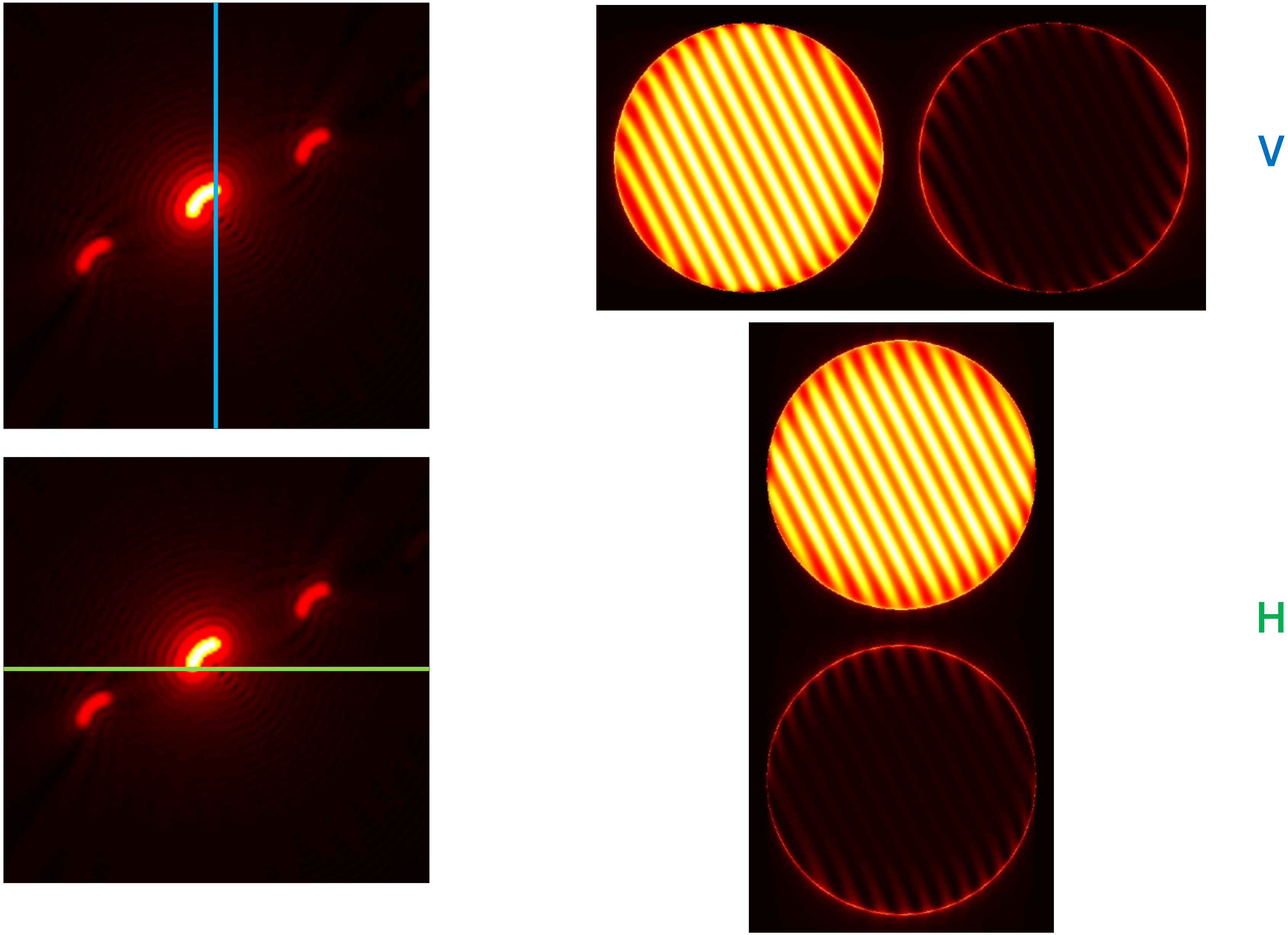
HO modes: Modulated Sharp Bi-O-edge



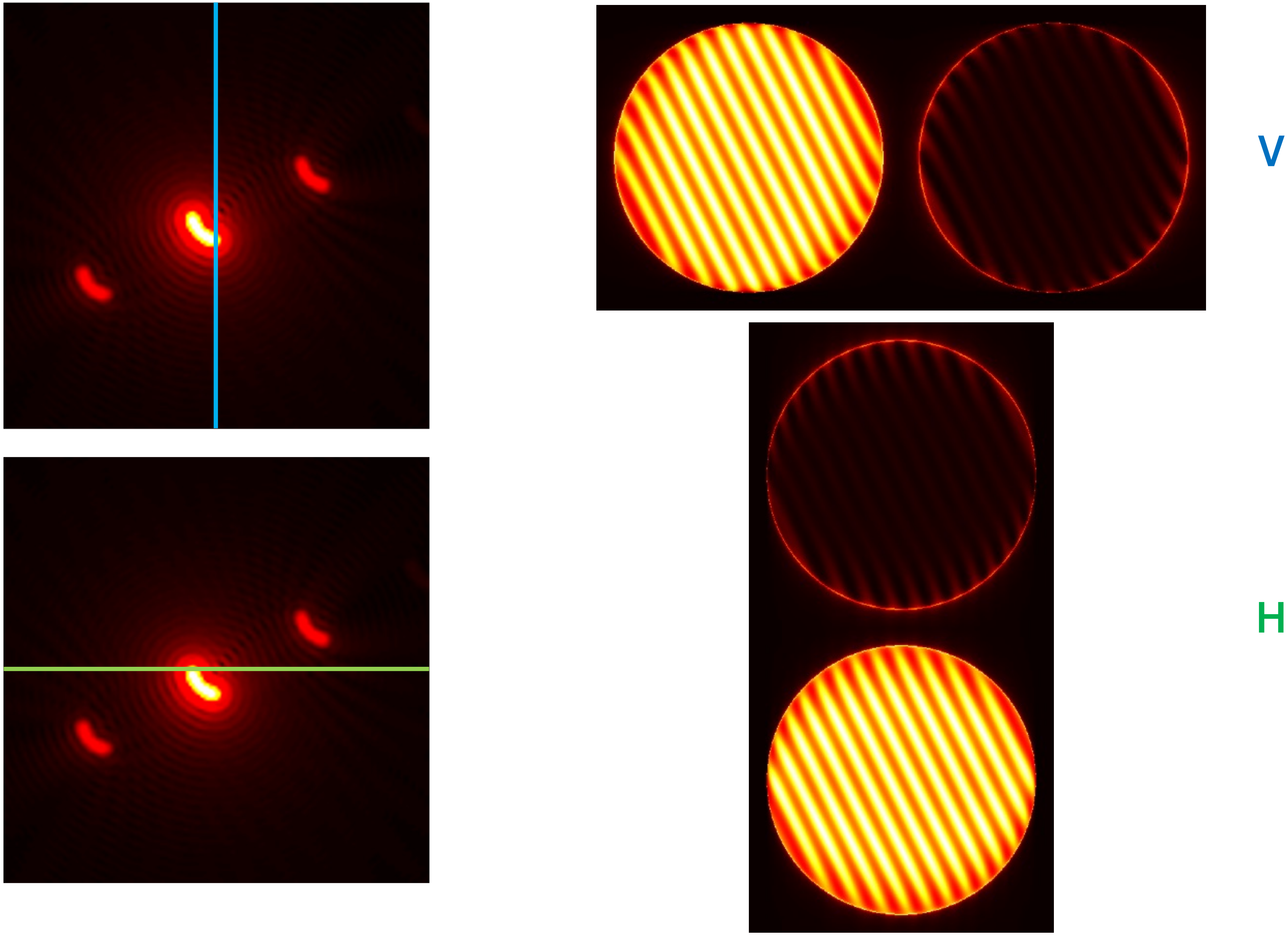
V

H

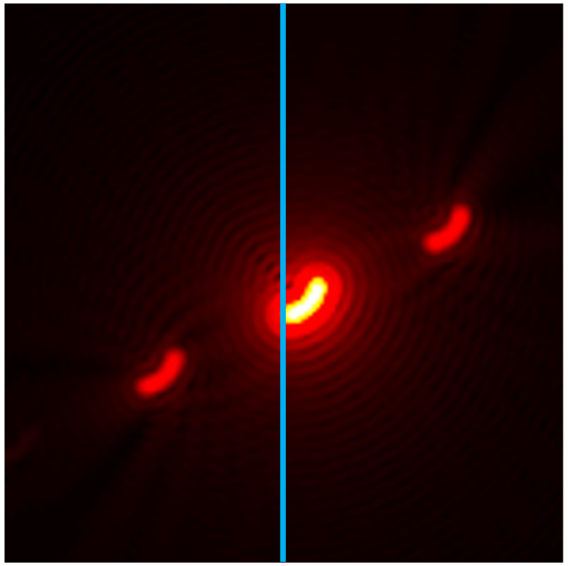
HO modes: Modulated Sharp Bi-O-edge

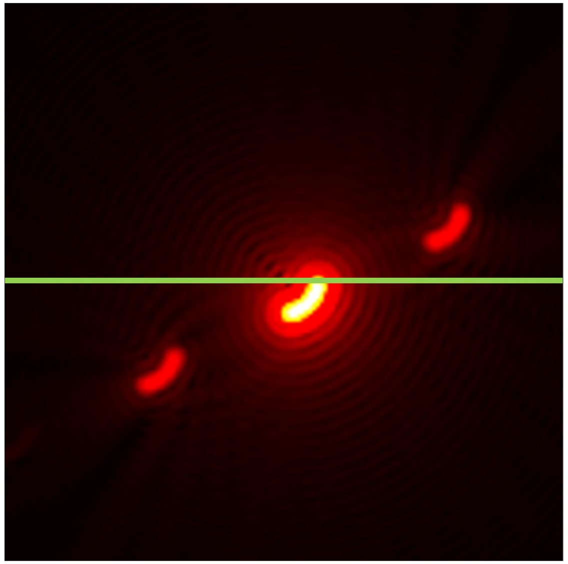


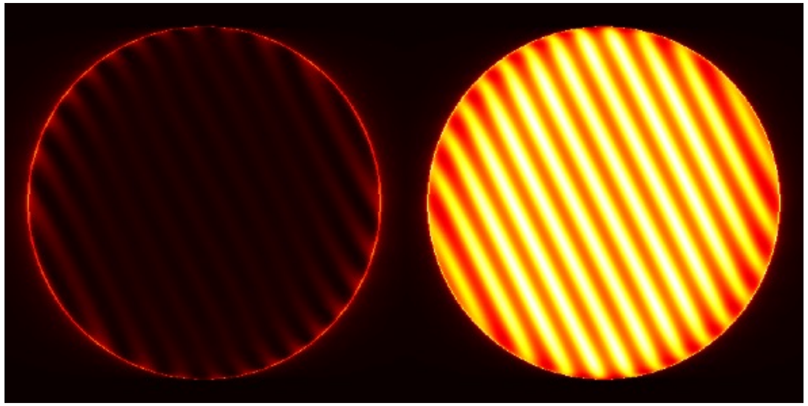
HO modes: Modulated Sharp Bi-O-edge



HO modes: Modulated Sharp Bi-O-edge







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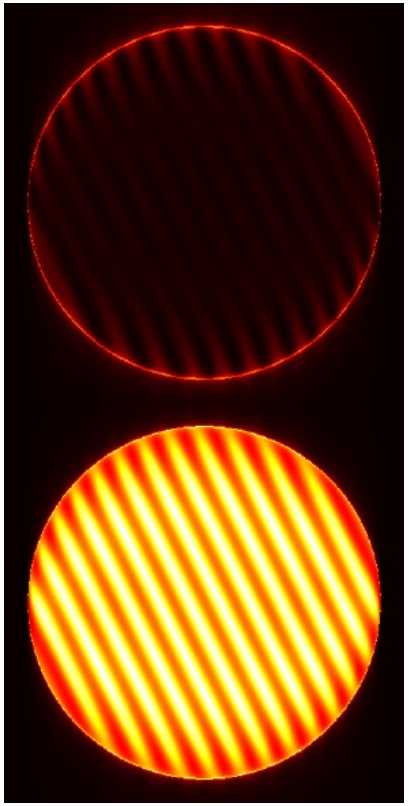

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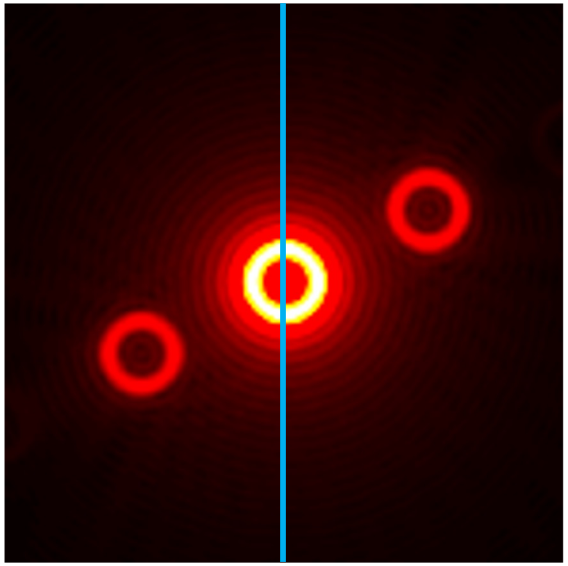

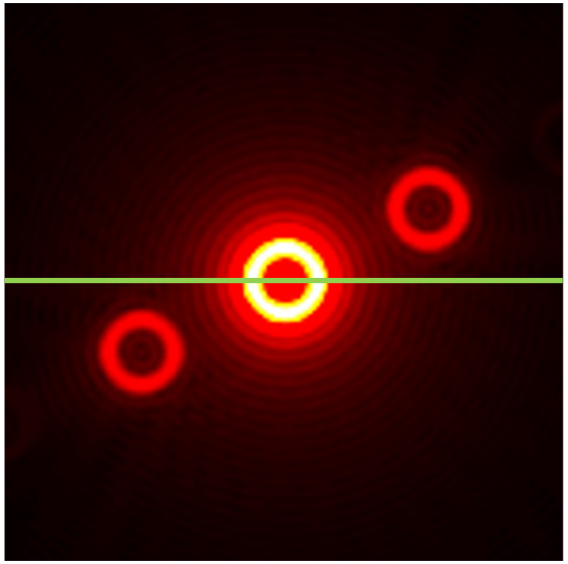

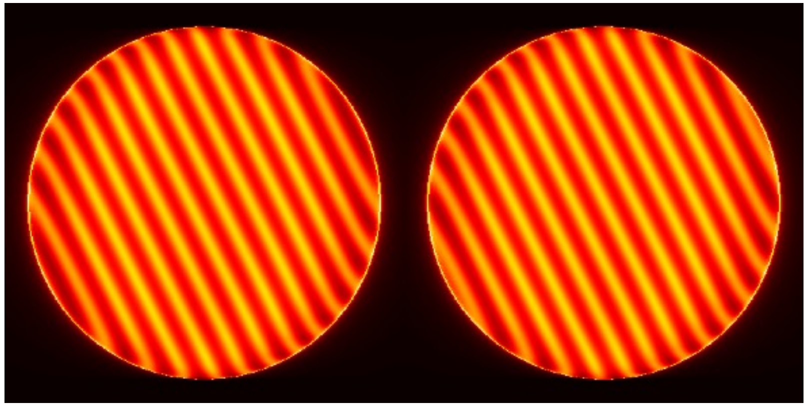

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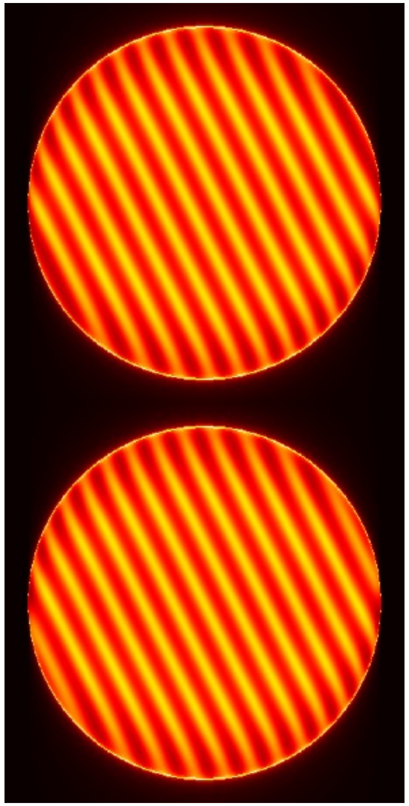


H

HO modes: Modulated Sharp Bi-O-edge

- Spatial Blindness per quadrant: 0%
- Modulation Duty Cycle per quadrant: 50%

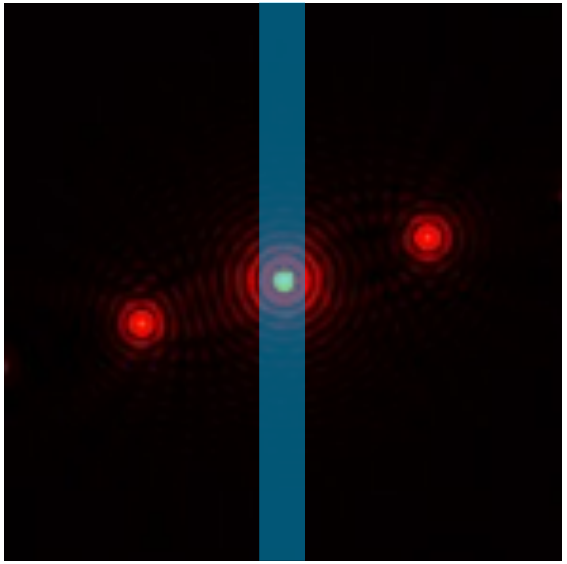
- → SENx = SENy = 2x0.5 = **1.0** → SEN^2=2.0

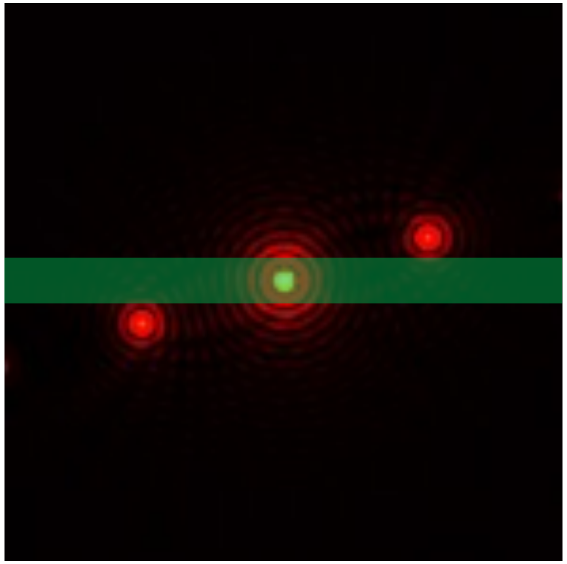
(diffraction by edges ignored)

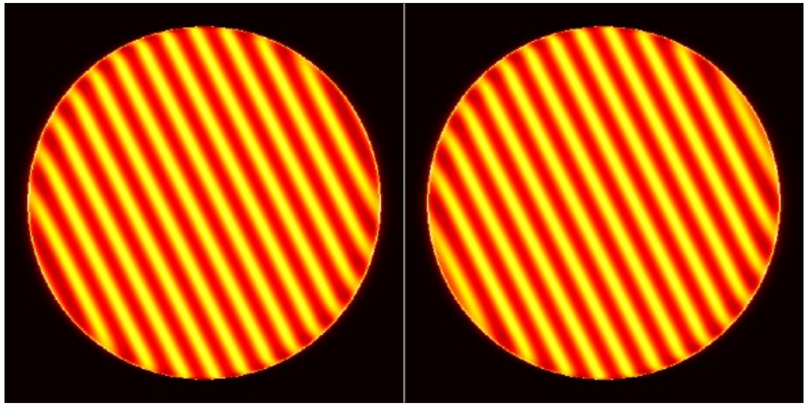
- Noise propagated on HO modes is:

$$\sigma^2_{ph,bio,HO} = \frac{1}{SEN^2_{bio,HO}} \sigma^2_{N,bio} = \frac{1}{2}\frac{2}{N} \longrightarrow \boxed{\sigma^2_{ph,bio,HO} = \frac{1}{N}}$$

Grey Bi-O-edge







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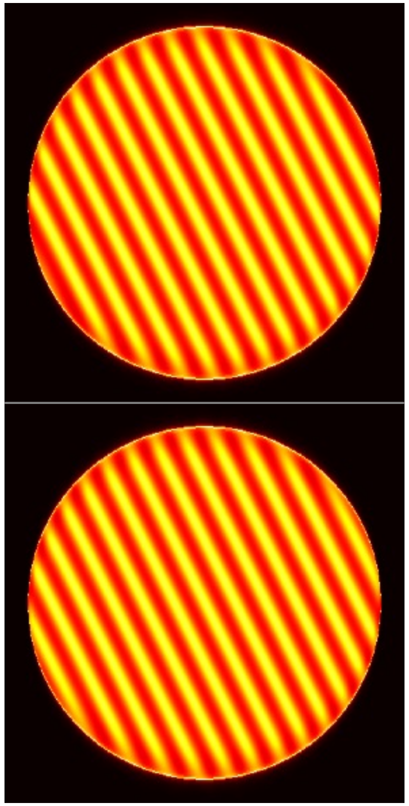

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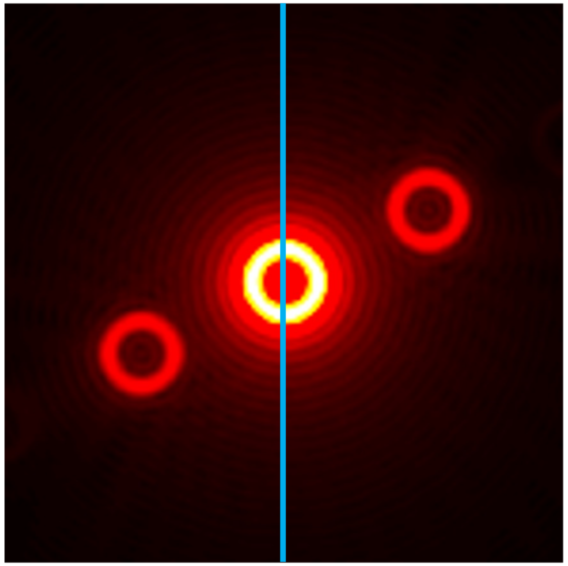
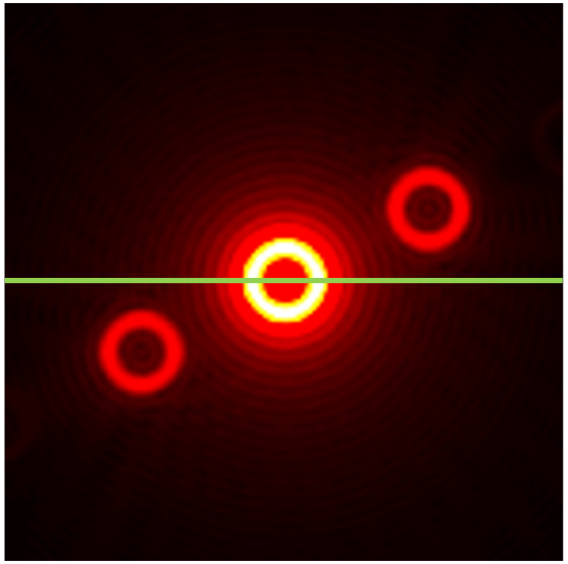
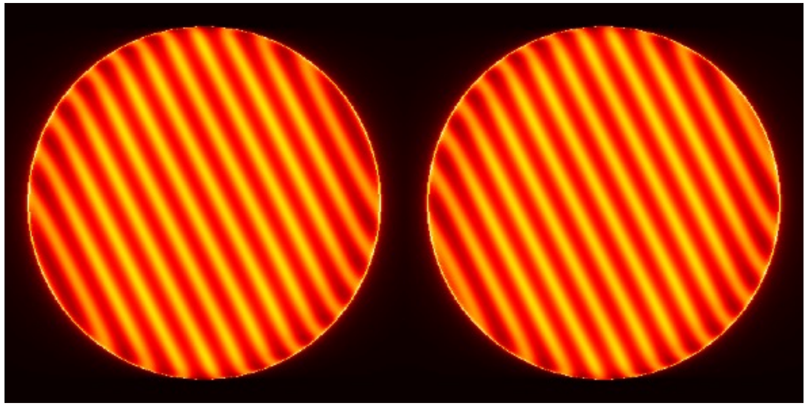
HO modes: Modulated Sharp Bi-O-edge







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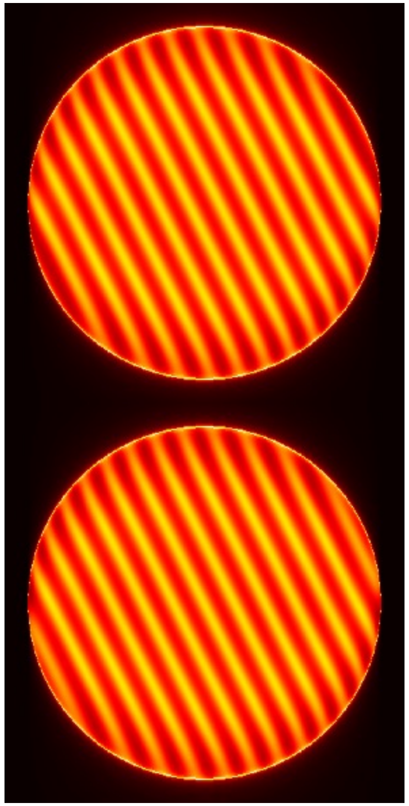


H

HO modes: Grey Bi-O-edge

- Spatial Blindness per quadrant: 0%
- (Modulation) Duty Cycle per quadrant: 100 %

- → SENx = SENy = √2 = → SEN^2=4.0

(diffraction by edges ignored)

- Noise propagated on HO modes is:

$$\sigma^2_{ph,grey,HO} = \frac{1}{SEN^2_{grey,HO}} \sigma^2_{N,bio} = \frac{1}{4}\frac{2}{N}$$

→

$$\sigma^2_{ph,grey,HO} = \frac{1}{2N}$$

Summary on noise propagation

- LO: Bi-O-edge (both) need twice more light than Pyramid
- HO: Sharp Bi-O-edge needs twice less light than Pyramid
- HO: Grey Bi-O-edge needs 4x less light than Pyramid

$$\sigma^2_{ph,bio,LO} = 2\,\sigma^2_{ph,pyr,LO}$$

LO Loss of factor of 2

$$\sigma^2_{ph,pyr,HO} = \frac{2}{N}$$

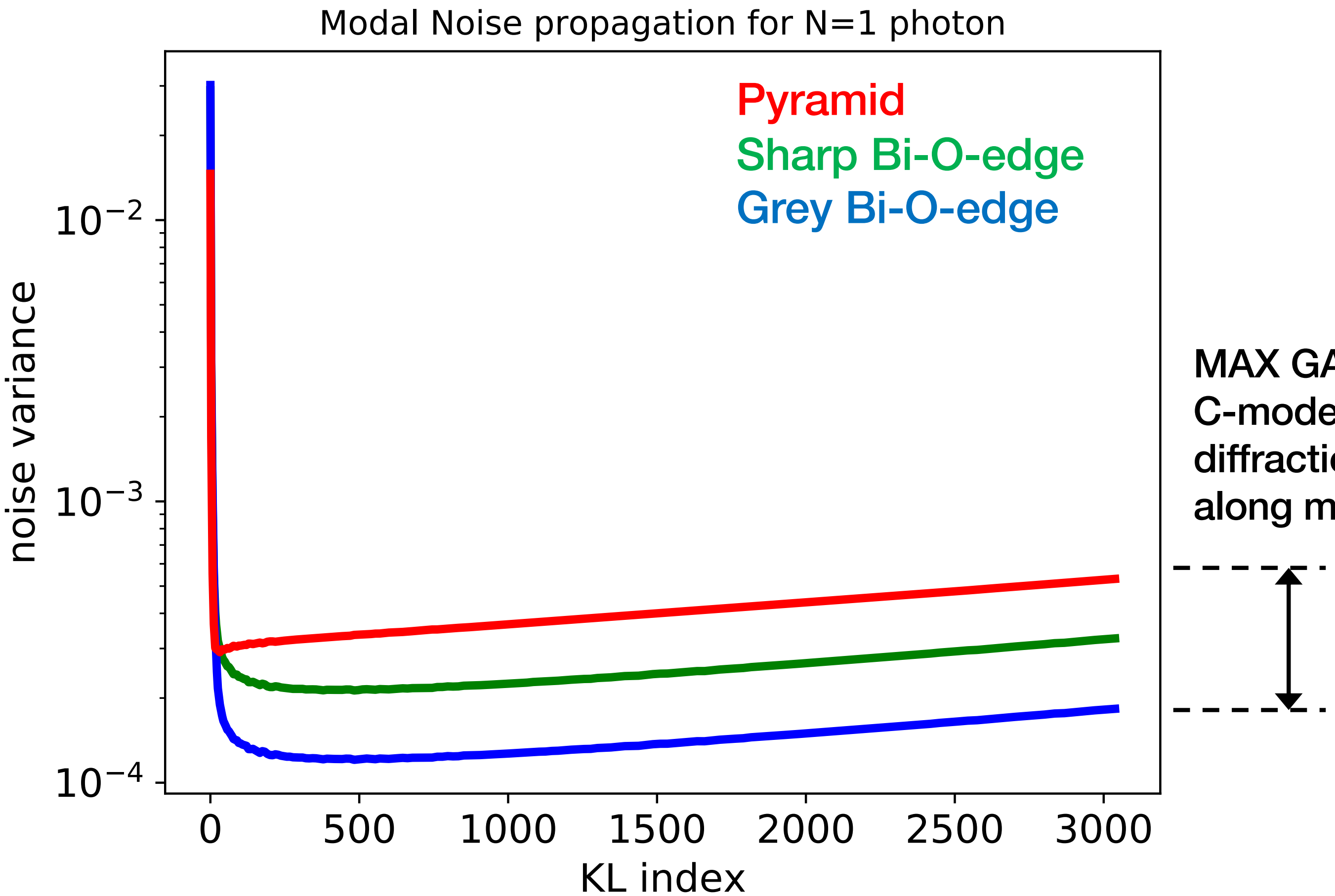
$$\sigma^2_{ph,bio,HO} = \frac{1}{N}$$

$$\sigma^2_{ph,grey,HO} = \frac{1}{2N}$$

HO MAX GAIN = 4

Modal Noise propagation

- SCAO/ELT: 38.5 m telescope, 3000 modes.
- C-model + KL index correspondence.
- WFS sampling 92x92, $r_{mod} = 2\,\lambda / \mathbf{D}$



MAX GAIN ≃3 (≠4)
C-model accounts for diffraction by edges along modulation path

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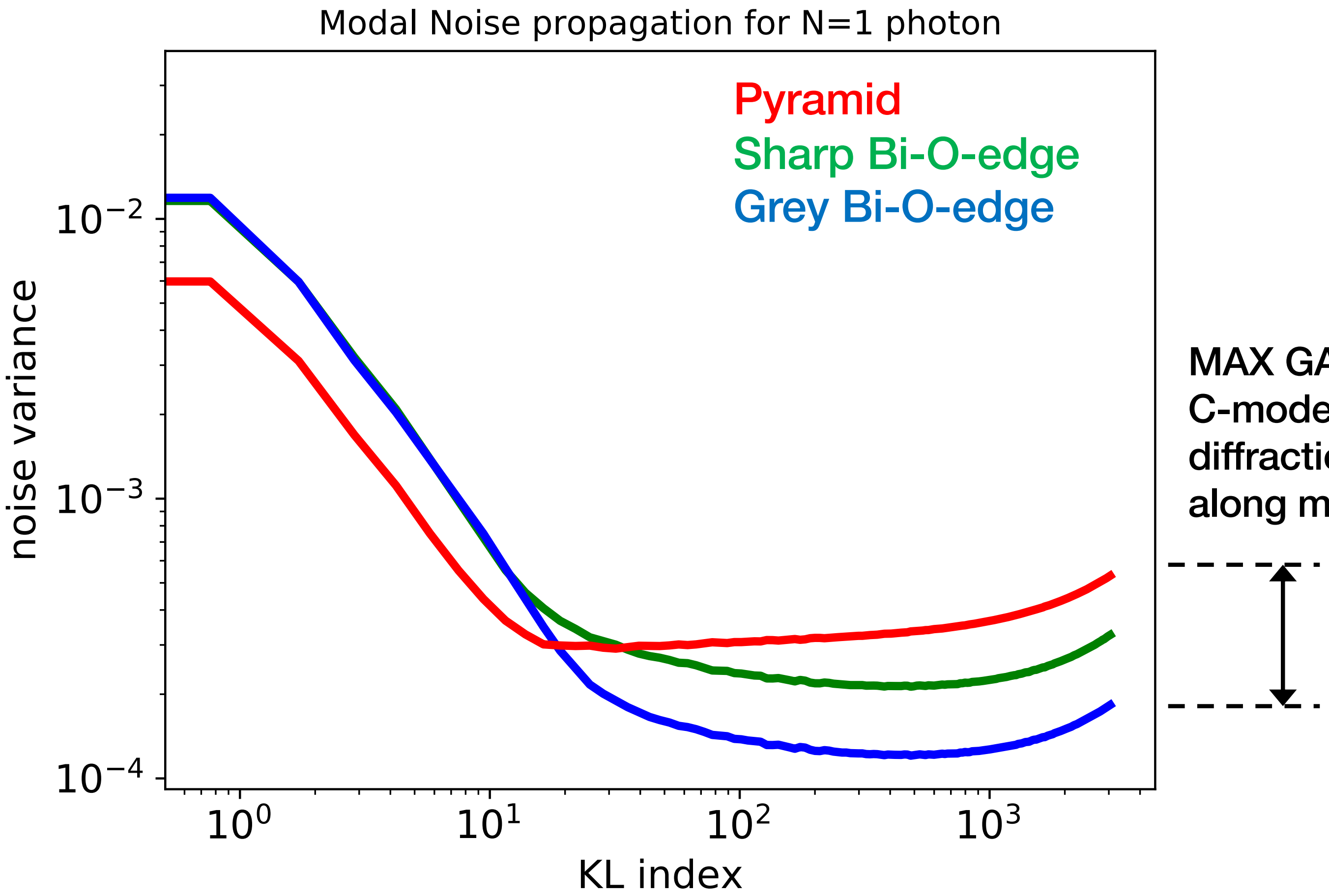
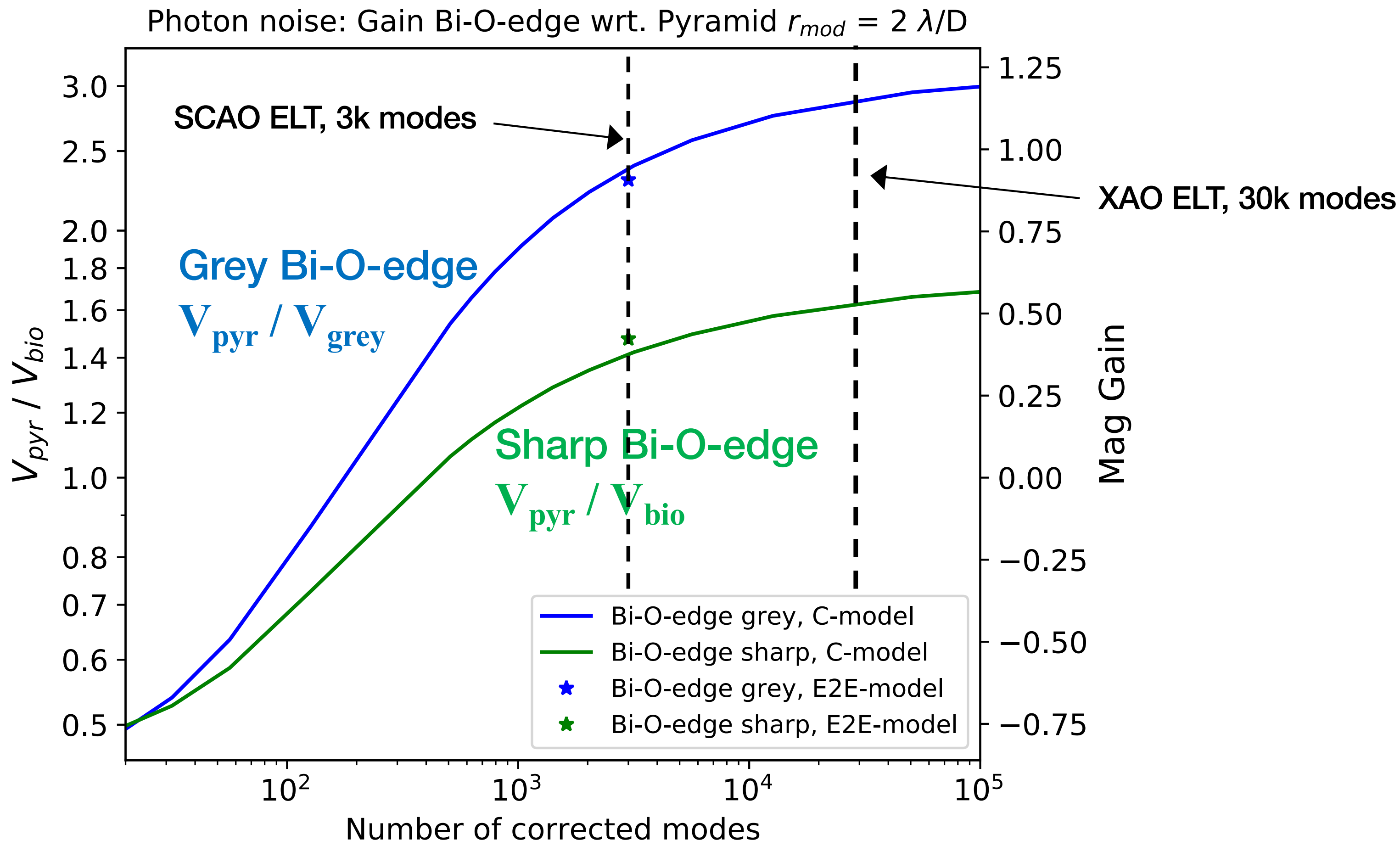


MAX GAIN ≃3 (≠4)
C-model accounts for diffraction by edges along modulation path

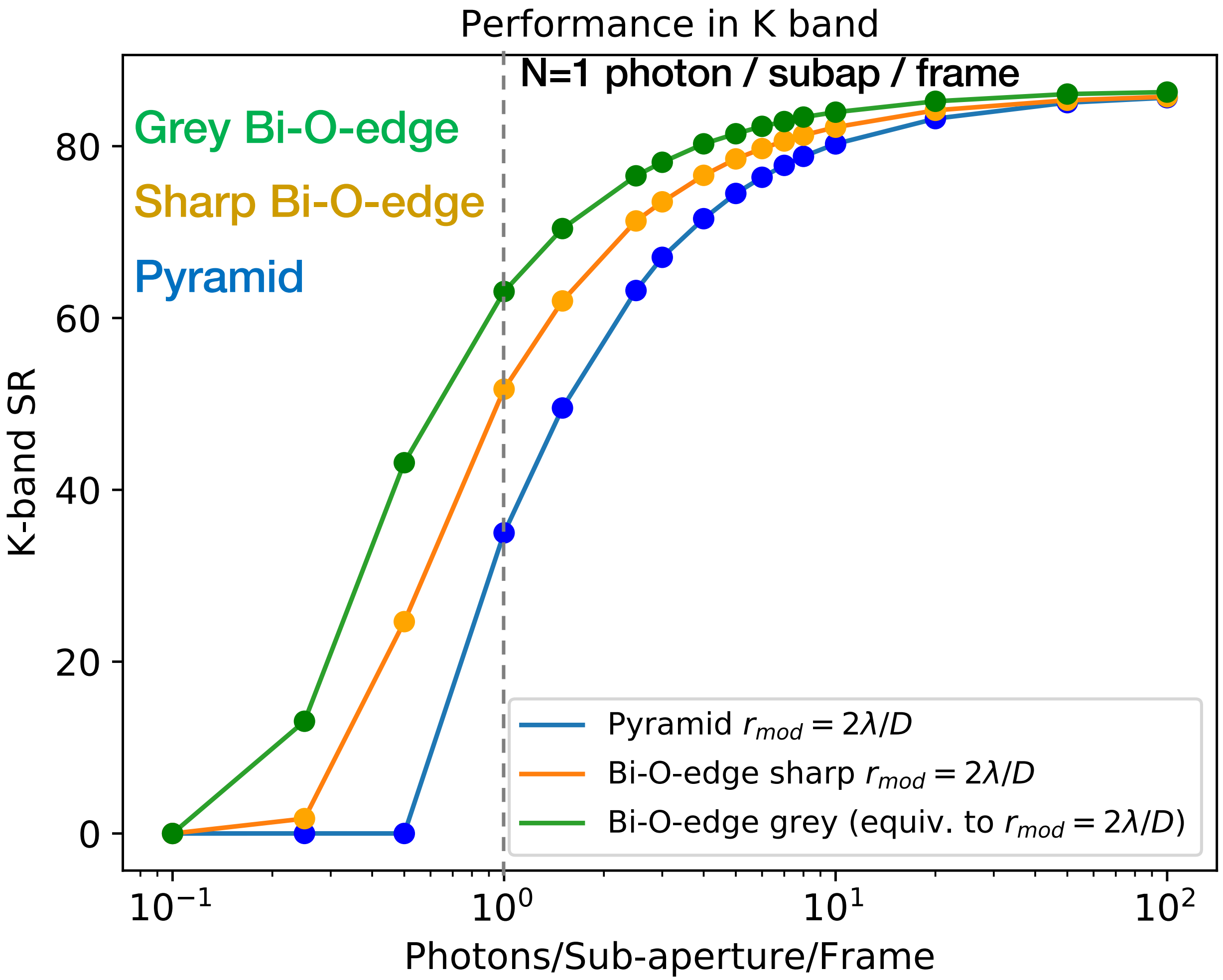
Gain wrt Pyramid in function of corrected modes

- 38.5 m telescope: analysis in function of # degrees of freedom
- Total Variance V: Bi-O GAIN in photon-efficiency = V_{pyr} / V_{bio}
- $r_{mod} = 2\ \lambda$ / **D**



Closed loop results with OOPAO

- Sensitivity gains are confirmed by simulations
- SCAO/ELT WFS sampling 92x92 3000 modes, K-band, $r_{mod} = 2$
- Median seeing. No gain optimization.

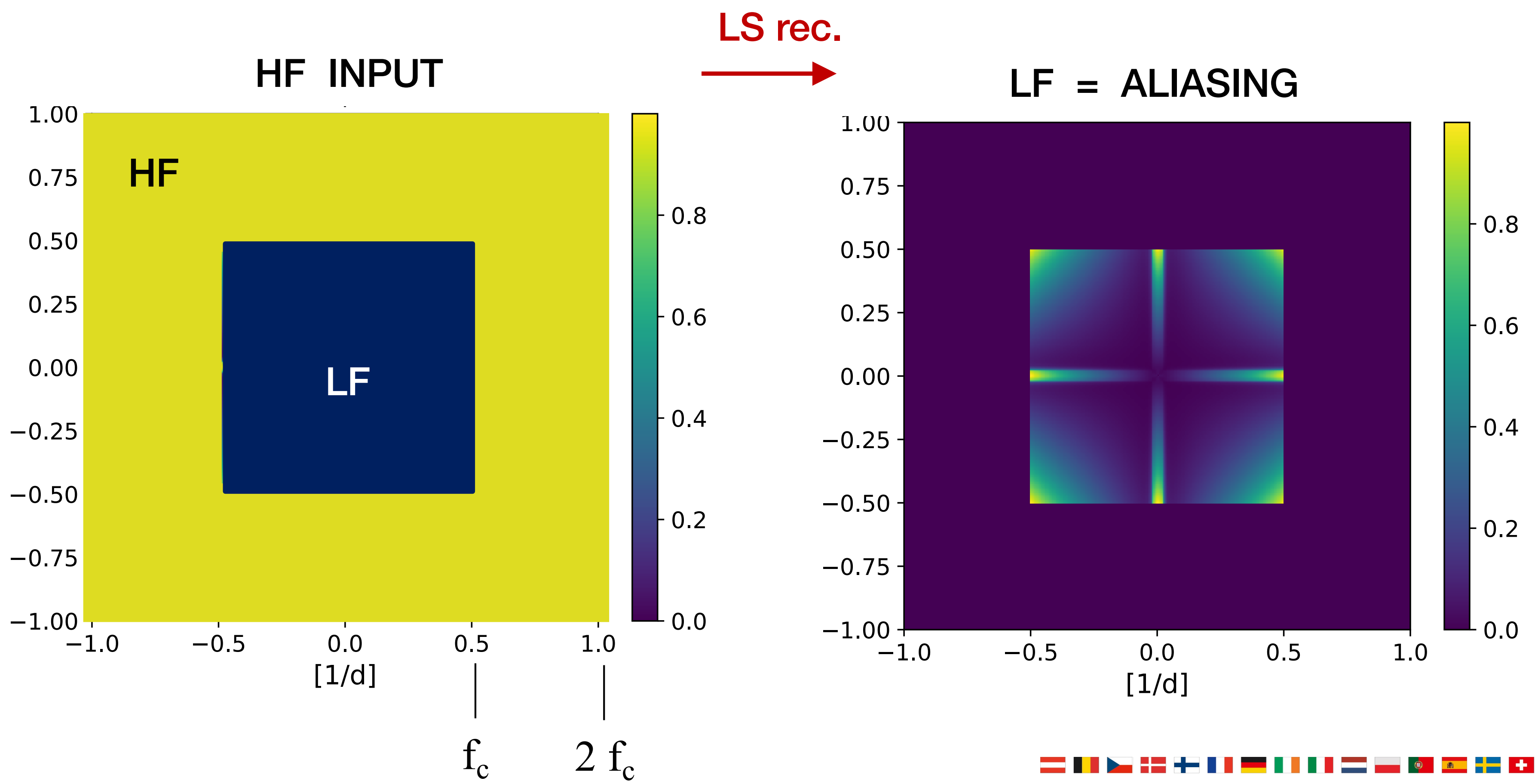


For N=1 photon / subap / frame

Grey Bi-O-edge: SR = 61%
Sharp Bi-O-edge: SR = 50%
Pyramid: SR = 32%

Super-Resolution: aliasing experiment

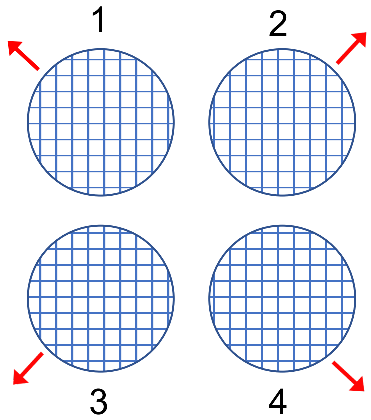
- d: sub-aperture size. $f_c = 1/(2d)$: AO cut-off frequency
- HF: $f_c < f < 2 f_c$ Input space in High Freq.
- LF: $f < f_c$ Least-Square Rec in Low Freq. space



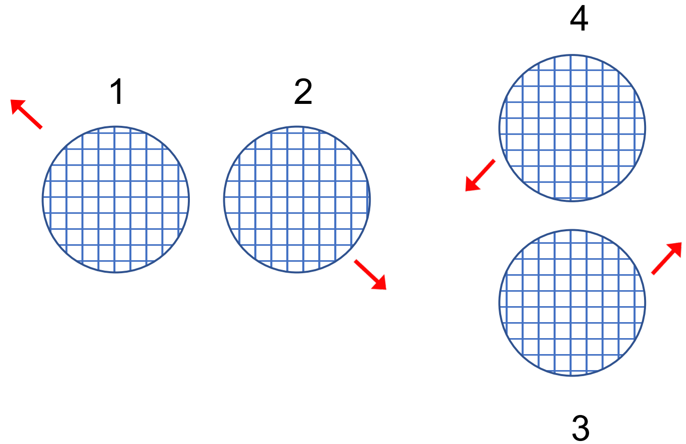
Super-Resolution

- X,Y shifts ± ¼ pixel: super-R up to 2 f_c (Oberti,2022)
- Full-pixel meta-intensity

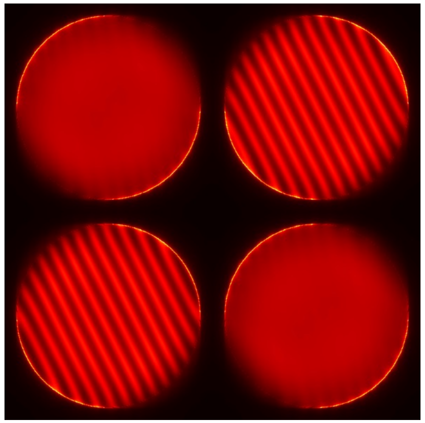
Pyramid

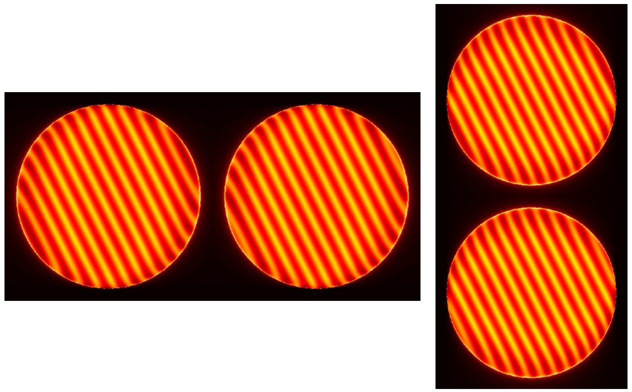


Bi-O-edge



Pyramid HO:
Only 2 quadrants have same info = HILBERT TRANSFORM

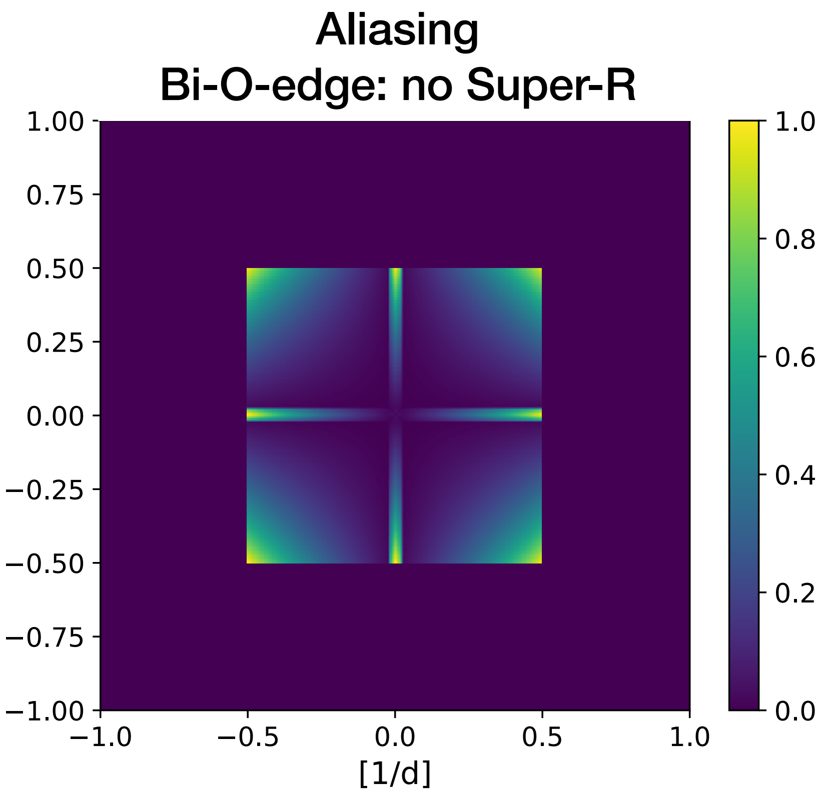


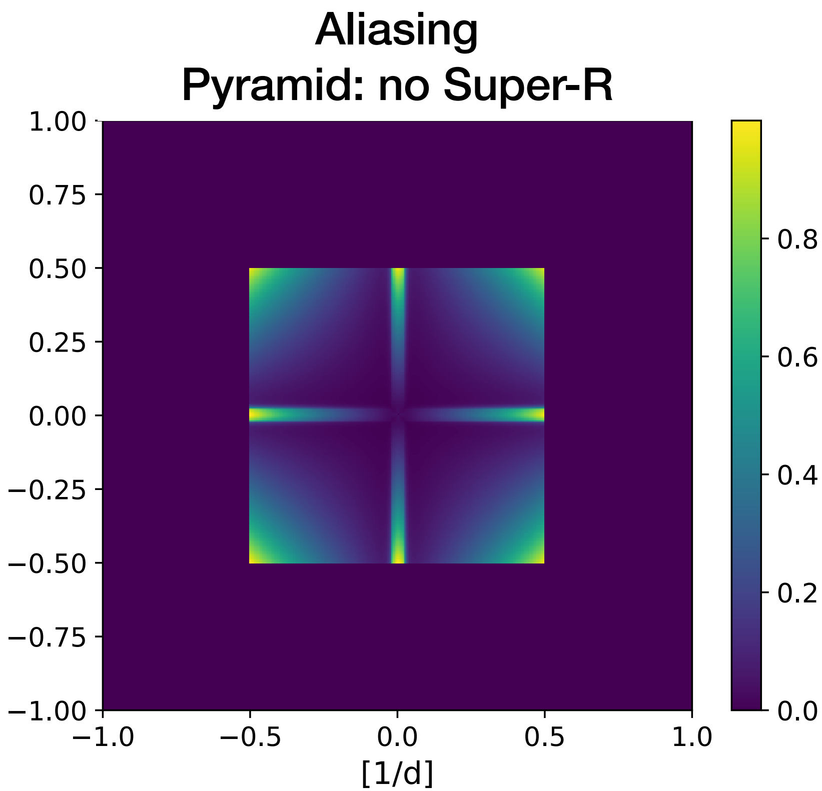


Bi-O-edge HO:
All 4 quadrants same info = HILBERT TRANSFORM

Aliasing (no Super-Resolution)

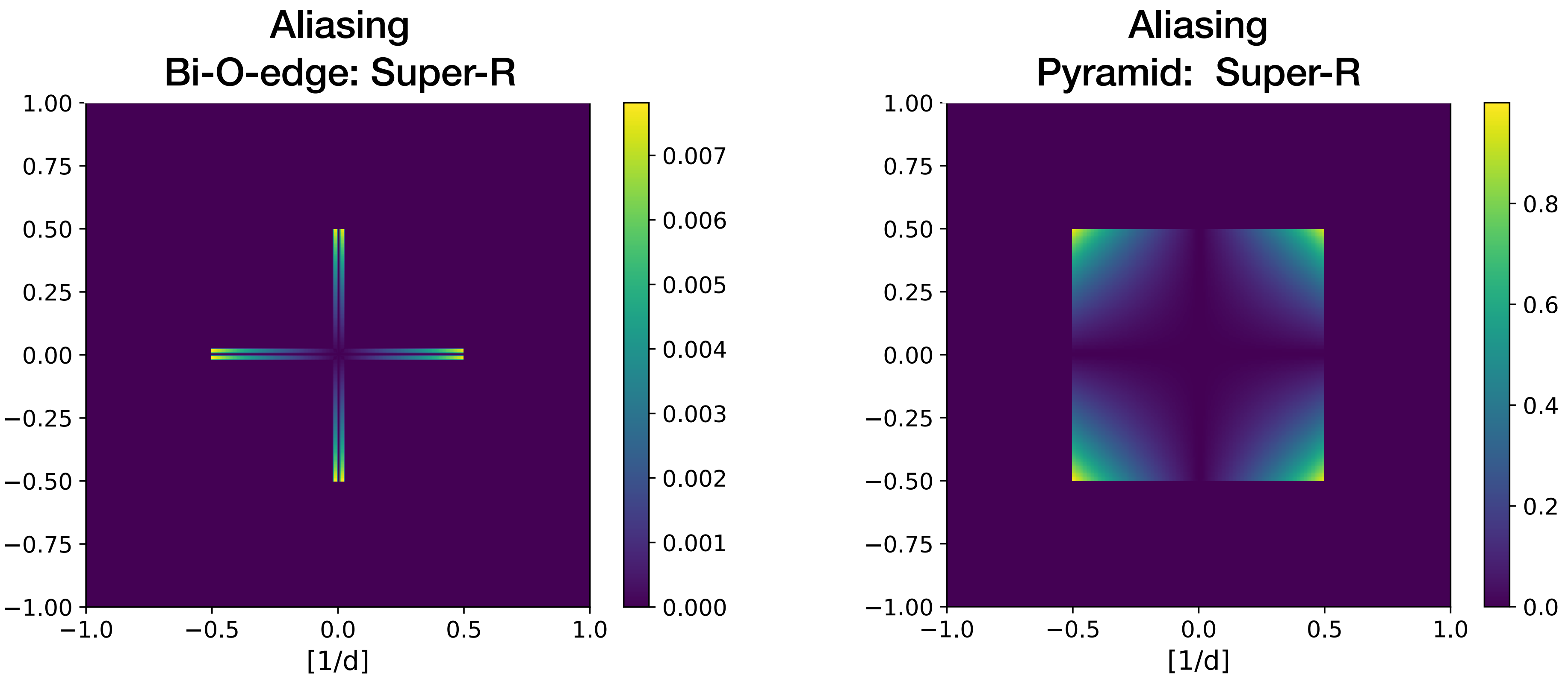
- Pyramid and Bi-O-edge have very similar aliasing in normal configuration





Aliasing with Super-Resolution

- Bi-O-edge: HO modes aliasing completely rejected
- Pyramid: Rejection of cross-like feature



Conclusions

- Bi-O-edge is a new class of FFWFS with same range as modulated pyramid but with up to 3 times better photon efficiency (paper in prep)

- Grey Bi-O-edge : static modulation → 2x more efficiency than dyn. mod.

overall gain wrt Pyramid for XAO on the ELT is ~1.2 mag

- Bi-O-edge : very efficient Super-Resolution → smaller modulation
  - Will boost non-linear reconstruction (Hutterer, 2019+…) (Nousiainen, 2022),
    - Cf. HO is responsible for most of non-linearity
  - Will ease trade-off # modes / # pixels / frame rate

- PCS: Bi-O-edge can extend the TLR of current concept:
  - R band observation: Higher SR for search of bio-signatures
  - Higher limiting mag: increase the search volume by ~5